SHEAR BEHAVIOR OF REINFORCED CONCRETE BEAMS WITH A SMALL AMOUNT OF WEB REINFORCEMENT

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ABSTRACT

In 1995, Hyogoken-nambu Earthquake destroyed many of Shinkansen viaduct structures that were a very large column section, approximately 900 mm., and a small amount of web reinforcement. With respect to this evidence, the shear strength formula was clarified in size effect but it still does not clarify the superposition method when a small amount of web reinforcement is extremely employed. Following that, the aim of this research is based on shear behavior and superposition method $V_{cr}+V_s$ by intending on a minimal amount of web reinforcement. Accordingly, 4 reinforced concrete specimens with web reinforcement ratio equal 0.035%, 0.05%, 0.065%, and 0.08% were conducted under monotonic loading. Moreover, two measuring systems for aggregate interlocking and shear resisted by web reinforcement were designed to use in investigating shear resistance mechanism. Consequently, the experiment shows that shear carrying capacity of 3 smallest amount of web reinforcement is the same and the superposition method is safe to predict the shear carrying capacity of reinforced concrete beam with a small amount of web reinforcement as shear span ratio 3.0. Similarly, other design codes also show the same degree of safety as the superposition method is employed. Not only that, but also the result of aggregate interlocking shows the same amount of stress transfer across the crack since the larger crack width corresponding to larger sliding of crack surface possesses the same stress transfer as that of smaller crack width and sliding.

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CHAPTER 1

GENERAL INTRODUCTION

- General and Problem Definition
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GENERAL INTRODUCTION

1.1 GENERAL AND PROBLEM DEFINITION

Amount of stirrup has a direct relation on the behavior of reinforced concrete members of general structure, since the structures are possible to fail in brittle manner without any warning sign if the shear stress rides over the shear carrying capacity. Great example of shear failure is the collapse of super-structures during Great-Hanshin Earthquake, in 1995. With respect to that evident, many of viaducts structures constructed as a rigid-framed [1] were destructed as shown in Figure 1-1. According to the mentioned structure, the amount of stirrup was lightly used. Therefore, the following question has been asked by many researchers for such a long time that did the estimation of shear strength of those was miscalculated?

Figure.1_1: Collapsed Structure.

One of possible and discovered reason is the size effect of the structure members, which was introduced by Okamura [2]. With respect to this reason, the formula derived from the specimen in experiments, which are very tiny compare to the real structure, did not include some important factor. It can be illustrated that strength of small specimens in experiments are affected in crack propagation by the reinforcing bar, so called bond effect, which increased the fracture energy of concrete. Nevertheless, in actual, the effect of bond from the reinforcement is very tiny since the members cross-section is so large that crack propagation is not able to confine by bond effect of reinforcement. It used to clarify and accept this problem, size effect, by many researchers and the consideration in its was added as size effect term in the shear strength design formula.

Another possible problem but it does not clarify yet is the shear carrying capacity's design concept since the concept is used as the strength of concrete at either failure or shear crack load and sum up with shear resistance of web reinforcement. Why the shear strength design concept seems to be not enough, it is possible that a small amount of web reinforcement cannot maintain the shear strength resist by concrete to be the same up to yielding point of stirrup itself. The remaining and possible reason is going to study and describe in this research thesis.

1.1.1 Current Use of Stirrup

As mentioned, the important factors drives shear failure in super-structure during Kobe's earthquake was explored. One factor originally based on the past design code since it used to overestimate in size-effect. A related consideration is because Japan Society of Civil Engineering's design guide at the constructing period used allowable stress design concept, and it had required a suddenly increasing of stirrup if the allowable shear strength of concrete did not enough as Figure 1-2. Consequently, escaping from suddenly increasing of stirrup, enlarge section of member to increase the allowable shear stress in concrete was commonly done. In summary, large section and small amount of stirrup are the consequence of designing that finally destructed by the earthquake.

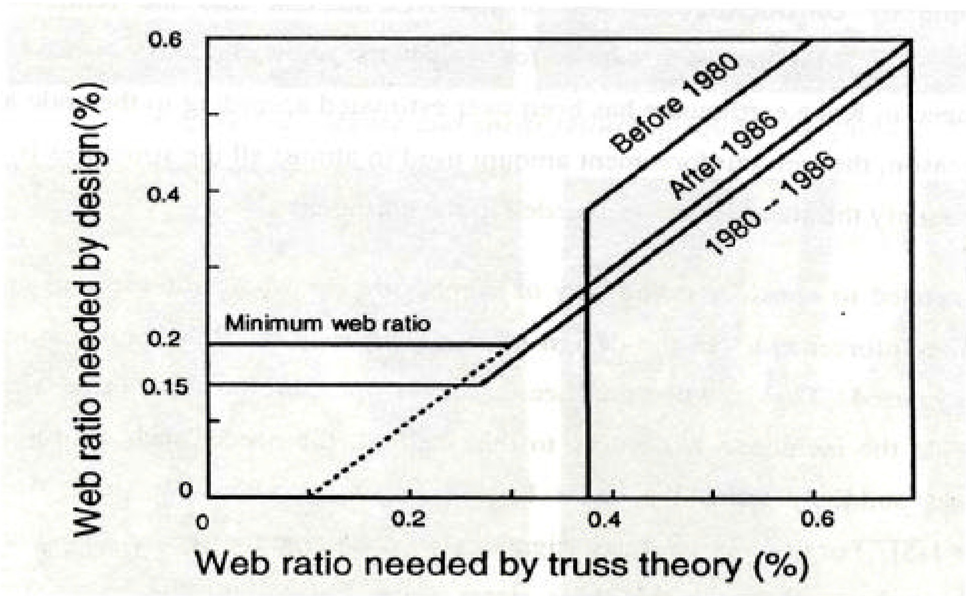


Figure.1_2: Evolution of Stirrup in Design Code.

To solve the problem, the amount of stirrup was later renewed and published by JSCE again in late year. Accordingly, the minimum requirement of stirrup was strictly assigned to 0.2% for the beneficial in seismic performances. Although, the designed structures follows the requirement have resulted in too much material used recently. In the last, it is possibly that the good shear strength prediction method for whole range of stirrup can reduce that of necessary cost.

1.1.2 Generality and Applicable of Superimposition Method

By considering simplicity, the several design guides stated that the shear carrying capacity can be determined by adding the contribution of shear carrying capacity of concrete with that of stirrup. Whatever the shear strength of concrete are proposed in many design guide, they are assumed that shear carrying capacity of concrete remains the same up to the yielding of stirrups if the minimum requirement of stirrup is provided. As it was claimed about small amount of web reinforcement, this is possible that the shear carrying capacity of concrete can not remain constant until the yielding of stirrup, and again this will result in ineffective calculation by $V_c + V_s$ as shown in Figure1-3.

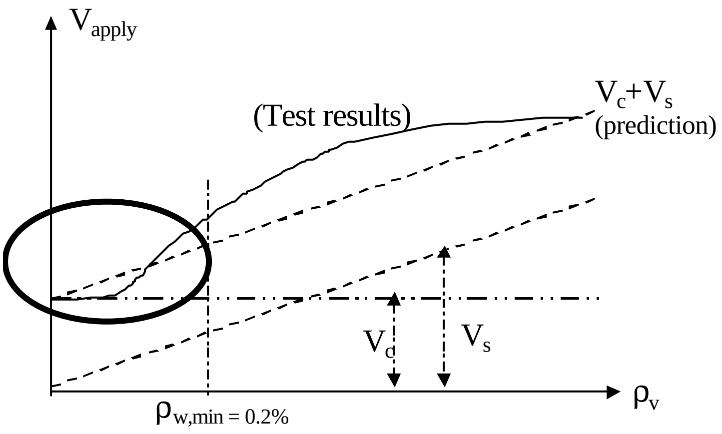


Figure.1_3: Assumption on Load Carrying Capacity by Varying Stirrup.

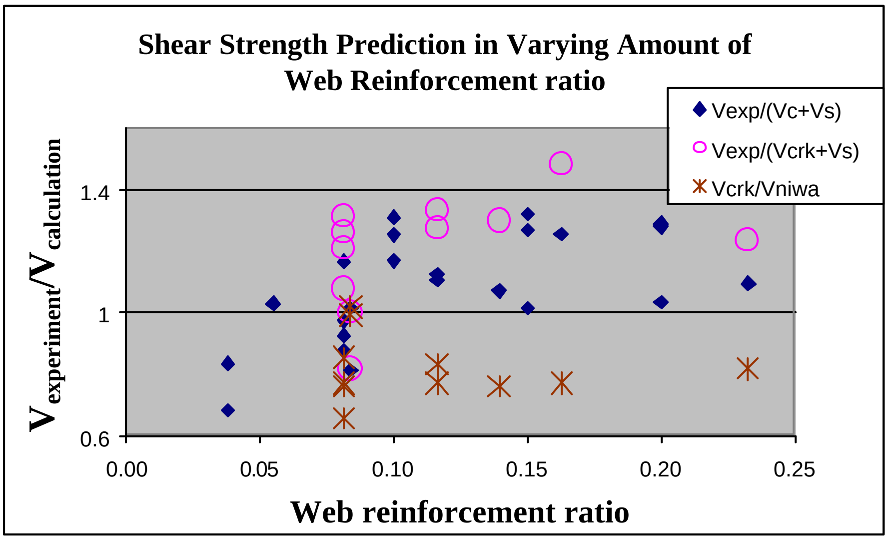


Figure.1_4: Prediction of shear strength by superposition method by varying web ratio.

To clarify and preliminary study, Figure.1_4 is the plot along the web reinforcement ratio, x-axis, by comparing the shear strength calculated by Niwa's equation [3], and the assumption of 45 degrees crack to the member axis with the experimental. The data used in the plotted graph are the large beams that tested in monotonic 3 and 4 point bending with depth higher than 500 mm., and contain quite small until very small amount of stirrup [4]-[9]. Beyond the web reinforcement ratio about 0.08%, it is clearly observed that the superposition method is safe enough to determine the shear carrying capacity. Although it is seemingly that the estimation by superposition method is not good enough. In summarize, the assumption and real experimental data, even they were rarely discovered in research publications, are given us reasonable idea that the shear strength of reinforced concrete beam with a small amount of web reinforcement can not simplicity determine by superposition method.

1.2 LITERATURE REVIEW

1.2.1 Mechanical and Determining of Shear resistance

Generally, as in Figure.1_5, the shear resisting mechanism of reinforced concrete can be qualitatively classified upon their behaviors as; 1.) Shear stress of concrete in compression zone; 2.) Aggregate interlocking along the crack plane; 3.) Dowel action of longitudinal reinforcement; 4.) Stirrup. Although, it is difficult to quantify the quantitative model in estimating shear resistance of concrete part item by item.

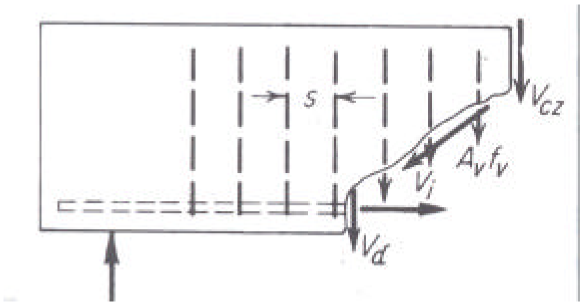


Figure.1_5: General Mechanical of Shear Resistance.

By considering that difficulty, many researchers [3,10] proposed the model to calculate the shear crack load of reinforced concrete without web reinforcement at shear cracking level by relying on the collected experimental results. Therefore, the shear strength of reinforced concrete beams with stirrup can be finally determined by adding that of concrete and stirrup based on 45-degree truss analogy. Those shear crack load can be determined by the following equation;

Okamura's Equation: $$V_{cr} = 0.20 f_c^{'1/3}(0.75 + 1.4\, d/a)(1 + b_p + b_d) \qquad \text{1-1}$$

$$b_p = \sqrt{p_w} - 1$$

$$b_d = d^{-1/4} - 1$$

Niwa's Equation: $$V_{cr} = 0.2\left(p_w f_c^{'}\right)^{1/3} d^{-1/4}(0.75 + 1.4\, d/a) \qquad \text{1-2}$$

Where,	$f_c^{'}$	:	compressive strength
	a	:	shear span length
	d	:	effective depth
	p_w	:	tensile reinforcement ratio

In somewhat advance concept, both concrete and stirrup are recognized to possess some interaction between them. Those were known as beneficial to the beam action [11] by effectively increasing in; 1.) The dowel action, because of the support offered by stirrups to longitudinal bars; 2.) The strength of concrete tooth, due to an inclined compression field associated with truss mechanism; 3.) The aggregate interlock strength. With respect to this, it shown that stirrup effectiveness can be defined by 'stirrup effectiveness function' [12] as 1-3 and 1-4, which is also function of moment induced in beam action, M_b, as shown.

$$\Psi = kI_b + p \qquad \text{1-3}$$

$$I_b = M_b / M_{uc} \tag{1-4}$$

Where, k, p : constant
I_b : beam action index
M_b : moment contributed by beam action
M_{uc} : ultimate moment

Conceptually, it can be said that the amount of stirrup has the direct relations to the shear resistance by increasing the effectiveness of beam action. Accordingly, in this terminology, shear strength contributed by concrete was remained constant, no effect of stirrup effectiveness, but changed in term of shear resisted by stirrup for simplifying. Shear strength of beams with web reinforcement can be determined by equation 1-5

$$V_u = V_c + \Psi V_s \tag{1-5}$$

According to the advance concerning, the shear carrying capacity of reinforced concrete beam in concrete part after cracking should not be regard to be the same as just crack [13] but it should be changed. However, in that experiment, the amount of stirrup used as much more larger than the minimum requirement. Nevertheless, at least, the behaviors of it after cracking like either load resistance or crack opening and crack sliding, which control the load-deformation of itself, it mainly depends on the amount of stirrup.

In summary, the shear carrying capacity of reinforced concrete beam with web reinforcement does not simply sum up that of concrete and stirrup together as V_c+V_s, but it has to concern with the interaction between them, which actually depend on amount of web reinforcement.

1.2.2 Provision of Minimum Stirrup in Several Design Codes

As aforementioned, the shear carrying capacity of reinforced concrete beam with web reinforcement can be simply calculated by V_c+V_s, which the shear force resisted by stirrup has to provide almost minimum not less than the code specified below since it is assumed that shear force resisted by concrete will remain the same. The following requirement is the minimum requirement of stirrup according to many designing codes.

JSCE 1986 [14]

Seismic design code : $\rho_s = 0.20\%$

ACI 318-83 [15]

Normal strength concrete (Fc' < 69 Mpa)

$$A_v = \frac{0.33 b_w s}{f_y} \quad : \quad \rho_s = 0.08 - 0.13\%$$

ACI 318-89 [16]

High strength concrete (Fc' > 69 Mpa)

$$A_v = \frac{f_c^{'}}{35}\frac{(0.33 b_w s)}{f_y} \quad : \quad \rho_s > 0.08$$

CSA 84 (Canada standard association) [17]

$$A_v = \frac{0.35 b_w s}{f_y} \quad : \quad \rho_s = 0.08 - 0.13\%$$

CSA 94 [18]

$$A_v = 0.06\sqrt{f_c^{'}}\frac{b_w s}{f_y}$$

AASHTO (LRFD Bridge design specification 1994) [19]

$$A_v = 0.083\sqrt{f_c^{'}}\frac{b_w s}{f_y}$$

Krauthammer [20]

$$A_v = \frac{0.448 b_w s}{f_y} \quad : \quad \rho_s = 0.1 - 0.16\%$$

All of current codes same as that of ACI, CSA, and AASHTO, already provided the minimum requirement of stirrup for high strength concrete since the higher strength of concrete affect in more brittle behavior and less shear transfer. Nevertheless, the shear carrying capacity has some possibilities to be less than the calculation even the minimum stirrup was provided, this is because the shear carrying capacity of concrete are not exactly predicted, which clearly observes in the case of ACI code due to either size effect or lightly reinforcement [4].

1.2.3 Shear Strength of Lightly Web Reinforcment Used

In case that the shear strength has to examine corresponding to the less amount of stirrup than minimum requirement that guidance is not provided in any design code, Angelakos, Bentz, and Collins [4] suggested to estimate by using the interpolation method. This method is to interpolate by tracing the straight line between shear strength without web reinforcement and full AASHTO minimum amount of stirrups using as shown in Figure.1_6.

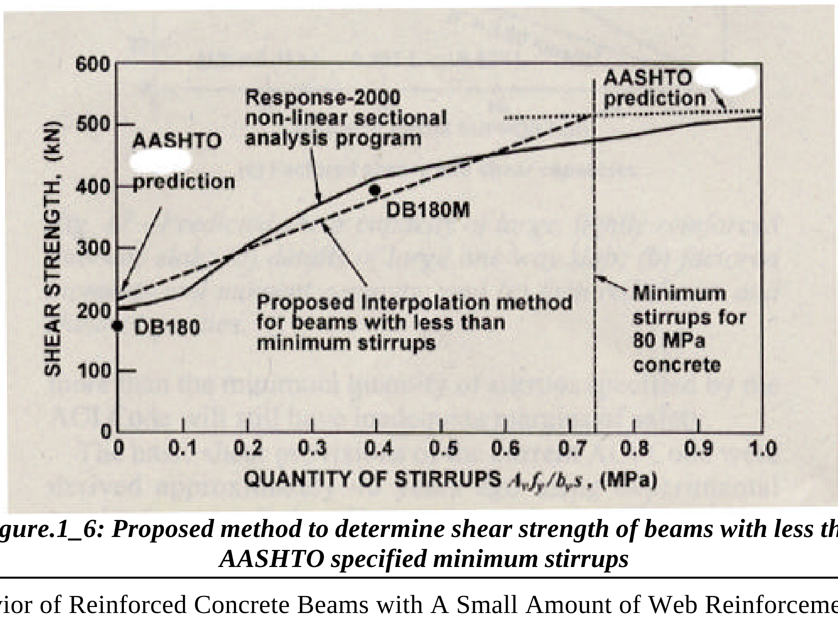


Figure.1_6: Proposed method to determine shear strength of beams with less than AASHTO specified minimum stirrups

1.3 RESEARCH OBJECTIVE

Nowadays, in Japan, the existing reinforced concrete structures used the stirrup at least equal to the minimum requirement specified in JSCE design guide; although, no theoretical insists that the amount of stirrup equals to 0.2% is appropriate, and how to determine shear carrying capacity as a small amount of web reinforcement used.

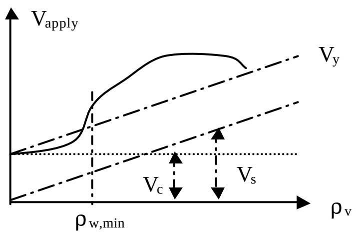
Up to this point of concerning, first objective of this study is to clarify the applicable of the superposition method that used in predicting shear carrying capacity of reinforced concrete beam with a small amount of web reinforcement at the shear span ratio 3.0. Parallel to the first objective, second objective is to make clear that what mechanisms take place when small amount of stirrup used. In summary, the great understanding of reinforced concrete member with a small amount of stirrup can be presented and hit it into shatter.

1.4 RESEARCH STRATEGY AND CONTENT

To perform the study in systematic and cover as much as details exist under several constrains, the topic in each chapter was designed to study in different levels but support to each other. Outline of research study is shown in Figure 1_7.

> ***Chapter 2****: This chapter is to illustrate experimental information, measuring systems, and data interpretation methods, which they will be stated in details as force transfer across shear crack and shear resisted by stirrup.*

Assumption



Experimental work

(Beam test under monotonic loading)

- With shear span to depth ratio equal to 3.

- Vary stirrup ratio at 0.03%, 0.05%, 0.06% and 0.08%.

Chapter 2
Experimental Investigation

- Specimen details.
- Measuring systems.
- Data interpretation methods

Chapter 3
Experimental Results

- Beams results.
- Strength comparisons.
- Wcomd Analysis results.

Chapter 4
Mechanism in Shear Resistance

- Mechanism of stress transfer

Figure.1_7: Research strategy.

> ***Chapter 3****: The experimental result corresponding with observation during experimental work will be noticed. The results of load carrying capacity compared with theoretically calculating equation are shown with some degree of difference.*

Chapter 4: *Leading to whole understanding in behavior of reinforced concrete structure with a small amount of web reinforcement, this chapter is to investigate and explain the mechanical of shear resistance by the means of stress transferring across the crack and stress contributed by stirrups.*

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CHAPTER 2

EXPERIMENTAL DETAILS

- Specimens Detail
- Measuring System
- Experiment Procedures

EXPERIMENTAL DETAILS

2.1 SPECIMENS DETAIL

2.1.1 Material Properties

For designing the reinforced concrete specimens and collecting the necessary data, the material properties should be correctly known by conducting the material tests. Thus, all principal materials used in constructing the reinforced concrete specimens, which are steel bars and ordinary concrete, are tested and shown in following content.

Steel Reinforced Bars

The general properties of used steel bars, which all is deformed type but different in grade between longitudinal and stirrup, are shown in Table.2_1. In the analysis, the stress-strain curves of steel bars are necessary used as constitutive equation; therefore, D4 stress-strain relation determined by regression method are shown in Figure.2_1. The constitutive equation is easily to obtain by regression method since the D4 bar shown non-exist of yield plateau. Therefore, it is proper to determine yield strength of D4 bar by the 0.002 off set.

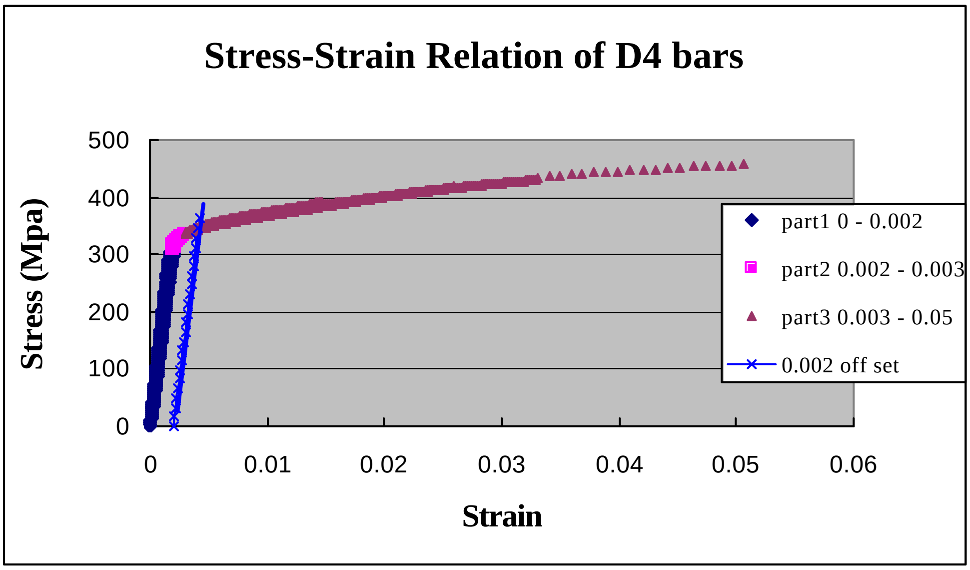


Figure.2_1: Stress-strain curve and Constitutive equation of D4 bars.

Table.2_1: General properties of used deformed bars.

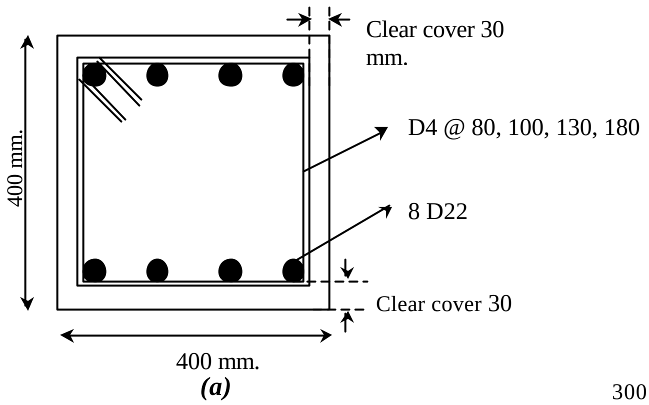
Steel Type	Nominal dia. (mm.)	Yield Strength (Mpa)	Ultimate Strength (Mpa)
D4 (SD 295)	4.0	350	457
D10 (SD 345)	10.0	391	586
D22 (USD 685)	22.0	718	985

Ordinary Concrete

The casted concrete of specimens is ordinary concrete with the compressive strength of 30 MPa at 28 days. Maximum aggregate size used as 20 mm. The curing method is done by pouring water on the foam sheet that covered the specimens for first seven days. Then, the beam specimens are cured by the air temperature in the laboratory.

2.1.2 Specimens Dimension and Parameters

According to the preliminary study in literature review, 4 reinforced concrete beams are designed and used in experiment with the same dimension but varying amount of web reinforcement to be less than 0.08%. As design is, the beams have unbalancing in shear span ratio, a/d, which they approximately equal to 3 in left span and 1.5 in right span. Respect to the target of the test, the right span has to highly reinforce with D10 bars to gain its ultimate strength higher than that of left span side. In Figure.2_2 *a* and *b*, the beams cross-section and scale down specimens layout are shown, respectively.



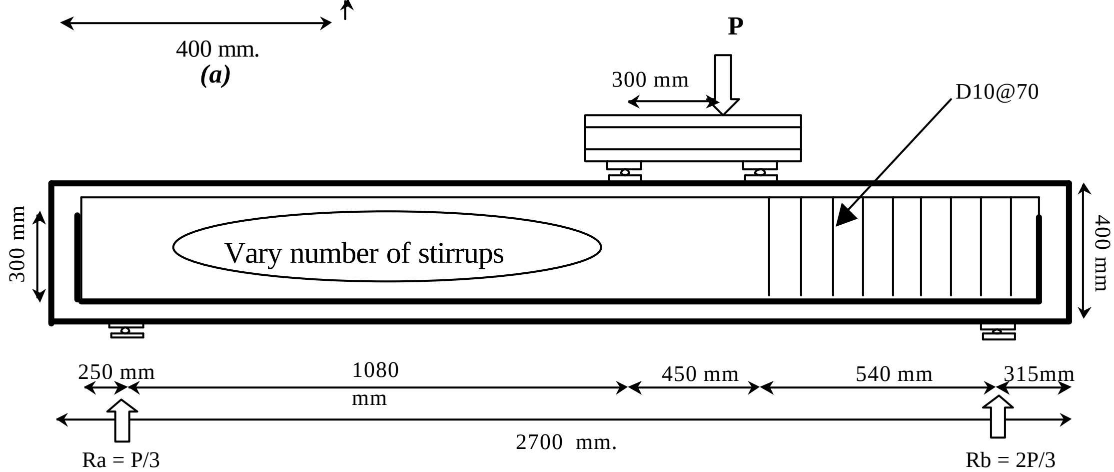


Figure.2_2: Tested beams layout.

Regarding to the beams layout and cross-section, the shear carrying capacity of each beams were determined according to Niwa's equation [1] parallel with superposition method, (V_c+V_s), and they also used the material properties from material experiments. The predicted shear strength of tested beams is shown in Table.2_2.

Table.2_2: Shear carrying capacity of each tested beams.

Beam NO.	Spacing of web Reinforcement (mm.)	V_s (kN)	V_c (Niwa) (kN)	V_u (kN)
1	D4 @ 80	40.58	160.81	226.6
2	D4 @ 100	32.47	163.35	187.8
3	D4 @ 130	24.97	161.06	190.8
4	D4 @ 180	18.04	166.6	187.5

2.2 MEASURING SYSTEMS

According to the research objectives, the mechanical of shear resistance have to be correctly investigated and considered in tested beams. Thus, the careful measurement methods have been designed according to the basis of shear resistance as the stress transfer across the crack and the stress of web reinforcement. Measuring systems have been desired on two microscopic models that can meaningfully explain local behaviors in the specified parts. Finally, it is possibly that mechanical of shear resistance in each two locations of beams can be accurately explained by these measuring systems by employing two microscopic models.

2.2.1 Measurement of Stress Transfer across Crack Plane

a) Theoretical Background:

Buja Bujadham proposed the model to calculate the stress across cracks concrete, so called Universal model for stress transfer across the crack [2,3], which it is actually the generalization of Bi Li's model, contact density model for stress transfer across cracks concrete [4]. The stress transfer across cracks concrete can be imaged and roughly illustrated in Figure.2_3. From figure, the opening and sliding of crack, *w* and τ, create the stress at contact unit of q_s, and summation of this stress at whole contact unit results in the stress transfer across the crack

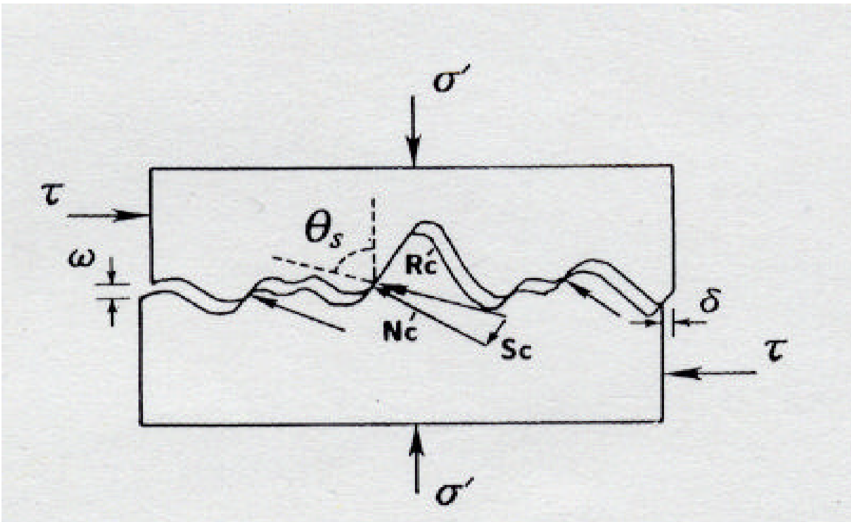


Figure.2_3: Stress transfer across the crack.

Contact density model concerns to the complex nature of crack surface, and then represents it in mathematical ways by using two proposals and three basic assumptions. Two basic proposals were crack geometry and contact stress direction that shown in Eq.2-1 and 2-2, respectively.

Area of a contact unit : $dA_q = A_t \Omega(q)dq$ 2-1

Contact stress direction : $q_s = q$ 2-2

Where, A_t : whole surface area per unit crack plane

$W(q)$: stochastic density function

q_s : resultant contact angle

q : contact angle before a unit deformed

Other three Eq.2-3, 2-4, and 2-5, were basic assumptions used in explain contact density function, elasto-plastic model for contact stress, and effective ratio of contact areas.

Contact density function: $\Omega(q) = 1/2\cos q$ 2-3

Elasto plastic model: $s_c' = R_s\left(w_q' - w_{qp}'\right)$ 2-4

Effective ratio of contact: $K(w) = 1 - \exp\left(1 - 0.5G_{\max}/w\right)$ 2-5

Where, w_q' : local deformation in normal direction of crack unit

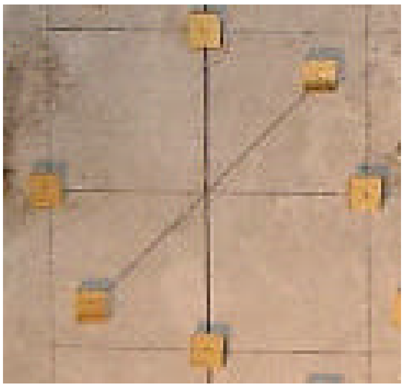
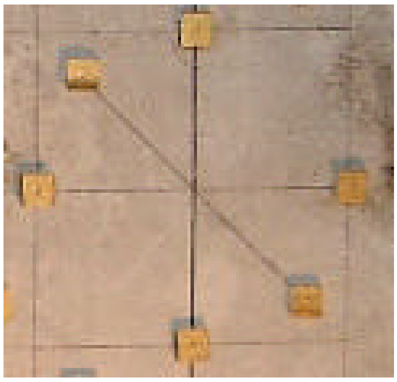
w_{qp}' : local plasticity in normal direction of crack unit

Regarding to these equations, the stress crosses cracks are able to calculate in somewhat specified loading paths, but not for complex loading paths same as mix-mode loading path, crack opening and sliding, of shear crack in beams. The unrealistic of contact stress direction proposal is the exclusion of frictional force, which is produced at the contact unit since it undergoes in mix mode. Therefore, application of normality rules is invalid to use in this research; comprising with the actual, anisotropic behavior should be considered instead.

Beside that of concerning, the effect generated from mix-mode loading path, the anisotropic plasiticity of mortar beneath aggregate, which generated from either the nature of contact angle or the loading directions, and its fracture are accounted in Universal model for stress transfer across cracks in concrete [3].

b) Measuring system

Contact extensometer and chips are designed to use in measuring the deformation of shear crack opening and sliding of tested beams. By utilizing coordinate transformations, the contact chips are located on the beam surface in both sides, north and south surfaced, at the longer span part, which has a/d = 3.0. Contact chips are cemented with the distance about 100 mm. apart from each others with the angle 0, 90, 135 degrees and 0,-45,-90 for north and south surfaces, respectively, as shown in Figure.2_4.

(a) Chips coordinate (north surface of beams). (b) Chips coordinate (south surface of beams).

(c). Chips on the beam surface.
Figure.2_4: Measuring system of stress across the cracks

By the proposed method, after crack taken place, the angle of crack at the origin point of contact chips coordinates are measured and utilized coordinate transformation equation, Eq.2-6, which has been well known used in strain rosette.

$$\begin{Bmatrix} x \\ y \\ xy \end{Bmatrix} = \begin{bmatrix} 1+\cos 2q_1 & 1-\cos 2q_1 & \sin 2q_1 \\ 1+\cos 2q_2 & 1-\cos 2q_2 & \sin 2q_2 \\ 1+\cos 2q_3 & 1-\cos 2q_3 & \sin 2q_3 \end{bmatrix} \begin{Bmatrix} a_1/2 \\ a_2/2 \\ a_3 \end{Bmatrix} \qquad 2\text{-}6$$

Where,

q_i	:	angle of *i* coordinate to tangential coordinate of crack
a_i	:	deformation respect to *i* coordinate
x	:	crack deformation in parallel direction of crack plane
y	:	crack deformation in vertical direction of crack plane
xy	:	crack deformation in tangential direction of crack plane

c) Data Interpretation

From the measurement, deformation in 0, 90, 135 and 0, -45, -90 degrees related with beam axis, and crack angle related to the beams axis are used in interpreting the crack displacement in lateral and horizontal direction, *y* and *xy* to the crack plane, respectively, as shown in the above equation.

Then, the universal model of stress transfer across the cracks is utilized to obtain the amount of stress and force resisted by the aggregate interlocking mechanism. The algorithm of model in calculating is shown in Figure.2_5. However, this outcome is still in the crack plane coordination and it need to convert to the beam coordinate again. By the coordinate transformation, the stress in Z direction, which is stress contribution in shear resistance process of aggregate interlocking mechanism, can be determined.

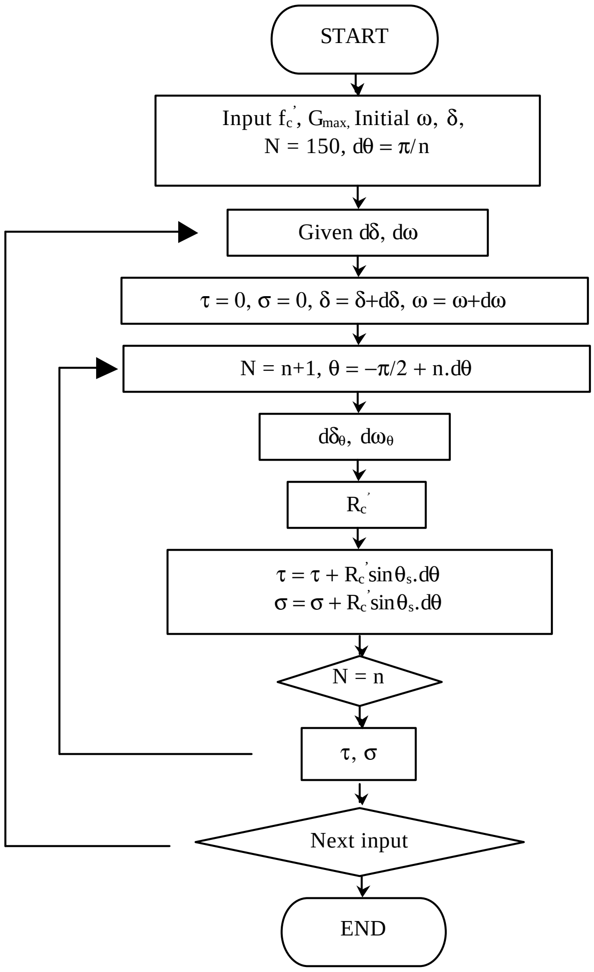
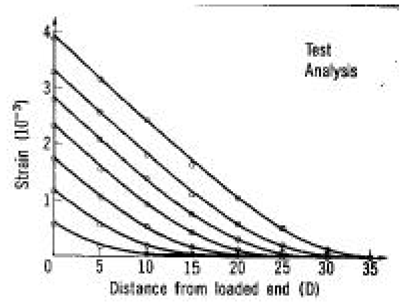


Figure.2_5: Algorithm of The Universal stress transfer across cracks concrete. [3]

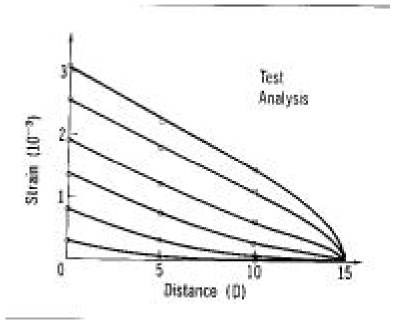
2.2.2 Measurement of Strain of Web Reinforcement at Crack Plane

a) Theoretical Background: Bond-Slip-Strain Relationship of Steel bar

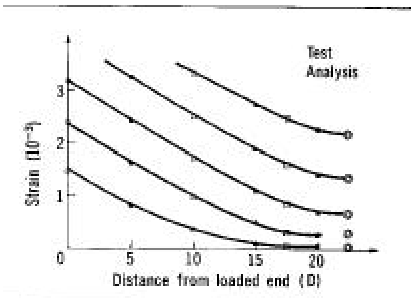
The tensile strain of reinforcement embedded in concrete has non-linear characteristic depends on the embedment length. This is because the present of bond between the reinforcing bar and surrounding concrete. With respect to the bond stress, the strain of steel bar in several cases is shown in Figure.2_6 *(a), (b),* and *(c),* which are bond of long embedment length, short embedment length, and bond of axial tension, respectively.



(a)



(b)



(c)

Figure.2_6: Strain of Embedded Reinforcing Bar. [5]

From the figures, the strain at specific point of reinforcing bar is nonlinearly reduced respect to the distance from the crack section. Moreover, it is not necessary that the same amount of slips have to be created by the same strain value, since the strain distribution of embedded bar is affected by boundary conditions, which are the embedment length, compressive strength of concrete, steel grade, and steel diameter.

Without considering the effect of strain value, many researchers failed to achieve model to determine bond stress. On the other hand, Shima [6], proposed a universal bond-slip-strain model for reinforced concrete, which take the effect of strain of embedded bar at the considered point. Bond-slip-strain model is shown in Eq.2-7.

$$\tau(\varepsilon, s) = \tau_o(s) g(\varepsilon) \qquad \textit{2-7}$$

Where, $\tau(\varepsilon,s)$ is local bond stress and $\tau_o(s)$ is intrinsic bond stress when strain is zero denoted by,

$$\tau_o(s) = f_c' k [\ln(1+5s)]^c \qquad \textit{2-8}$$

$$g(\varepsilon) = \frac{1}{1+10^5 \varepsilon} \qquad \textit{2-9}$$

Where,	$f_c^{'}$	:	compressive strength of concrete
	k	:	constant equal to 0.73
	c	:	constant equal to 3,
	s	:	non-dimensional slip equal to 1000S/d,
	S	:	slip
	d	:	diameter of steel bar
	ε	:	strain of steel bar

Nevertheless, this experiment does not intend to measure the bond stress but it intends to measure stress at the crack position. Fortunately, the strain at the crack position can be determined by utilizing bond stress equation.

b) Measuring method

By utilizing a set of equations corresponding with strain from two nearing positions along the embedded bar, stirrup, the strain at crack position can be indirectly calculated. The set of equations according to the local bond behavior are shown in Figure.2_8, which are bond-stress-slip equation, slip-strain relation, constitutive of steel bar, and equilibrium equation, respectively.

According to this method, the strains at two points are located at 3 cm. apart from each other. Furthermore, the position of strain gauges should be mouthed in the range 25 times of diameter from the crack section to ensure that the strain before yielding can be investigated. With respect to this method, the approximate crack positions should be determined in first place and it may be properly computed by some finite element program, so called Wcomd (2 dimensions, plate element). Therefore, the approximate solution by Wcomd is employed to locate the position of strain gauges at each stirrup.

Bond-slip-strain model: $t = t_o(s).g(e)$

Slip-strain relation: $S = \sum (e.\Delta x)$

Constitutive of steel bar: $s = s(e)$

Equilibrium equation: $\Delta s.A_s = \bar{t}.pd.\Delta x$

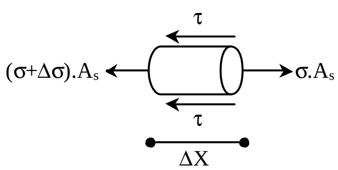


Figure.2_8: Equation for calculating strain of stirrup at crack plane.

Each reinforced concrete beam that analyzed by Wcomd shows almost the same crack position in tested span as the sample in Figure.2_9; therefore, the crack formation can aspect to be the same for simplicity in locating the strain gauge. The strain gauges positions in each stirrup leg of all beam are shown in Table.2_3, which the height are measured from the bottom of stirrup, the distances origin at the support, and the crack heights are estimated from 45-degree crack tracing from the loading point.

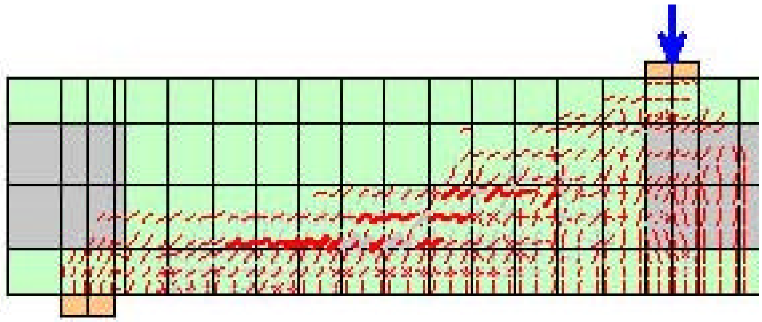


Figure.2_9: Crack pattern of tested beams.

Table.2_3: Stirrup strain position at each stirrup legs.

Specimen No.1 spacing 80 mm.

Position (mm)	Height1	Height2	Height3
1080	-	-	-
1000	346	316	286
920	266	236	206
840	206	236	266
760	126	156	186
680	66	96	126
600	66	96	126
520	66	96	
440	66	96	
360	66	96	
280	66	96	
200	66	96	
120	66	96	
40	66	96	

Specimen No.2 spacing 100 mm.

Position (mm)	Height1	Height2	Height3
1080	-	-	-
980	326	296	266
880	246	276	306
780	146	176	206
680	66	96	126
580	66	96	126
480	66	96	
380	66	96	
280	66	96	
180	66	96	
80	66	96	

Specimen No.3 spacing 130 mm.

Position (mm)	Height1	Height2	Height3
1080	-	-	-
950	296	266	236
820	186	216	246
690	66	96	126
560	66	96	126
430	66	96	26
300	66	96	26
170	66	96	26
40	66	96	26

Specimen No.4 spacing 180 mm.

Position (mm)	Height1	Height2	Height3
1080	-	-	-
900	266	296	326
720	86	116	146
540	66	96	126
360	66	96	
180	66	96	
0			

c) Data interpretation

From the measuring data, two strain values at two exact distances from the crack section are input in the computer algorithm for iteratively determining the slip of the farthest distance gauge. After the strain and slip of one exact location was exactly known, the extrapolating of strain distribution can be done by assuming strain in next position and then checking the equilibrium of stress between the bond and stress of embed bar in each step. The algorithm of calculating the strain distribution, slip, and stress at crack position is shown in Figure.2_10.

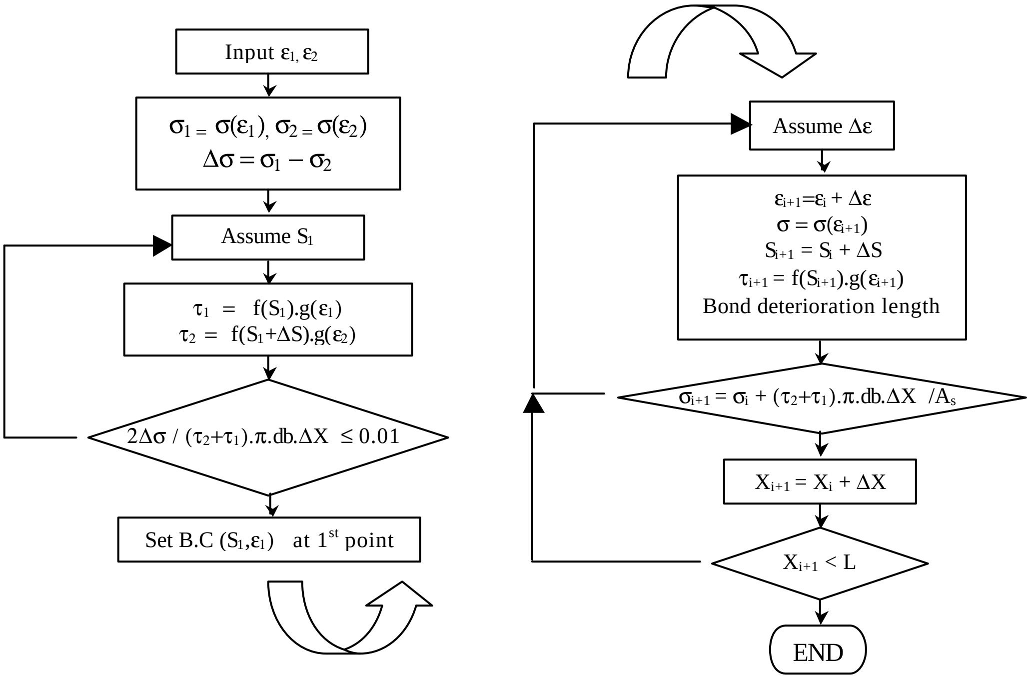


Figure.2_10: Algorithm for calculating strain of web reinforcement at crack plane.

2.3 EXPERIMENT PROCEDURES

Specimens were set up in the loading frame with hand-pumping loading system. As illustrated in Figure.2_11, the experiment set up before testing in each beam was shown. The loading method was a load controlling and the measuring can be read at the each loading steps equal to 30 kN. The load deflection can trace according to the load-measured values and deflection measured value from the load cell and CDP, respectively.

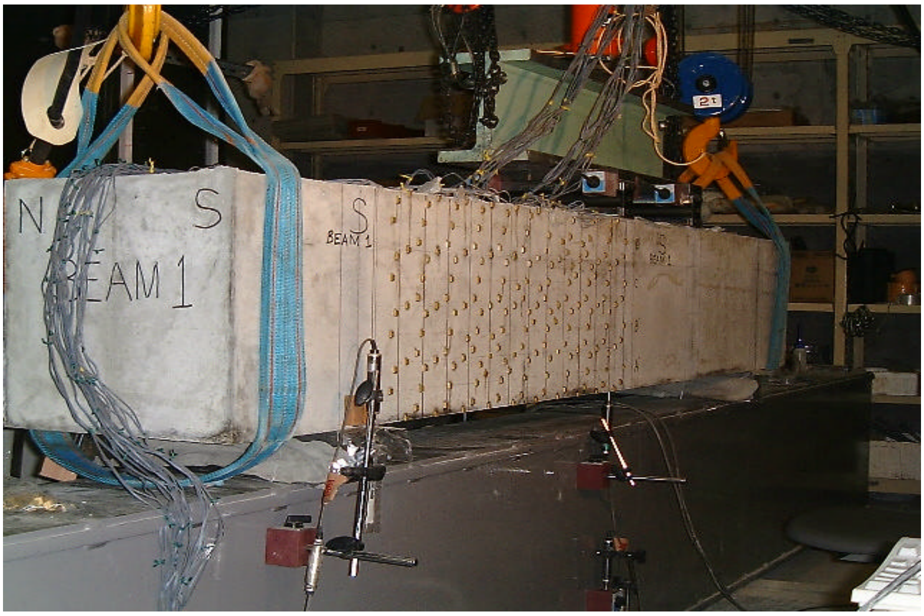


Figure.2_11: Specimen set up in the tested frame.

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CHAPTER 3

EXPERIMENTAL RESULTS AND INVESTIGATION

- Strength and Determination Comparison
- Discussion on Experimental Results

EXPERIMENTAL RESULTS AND INVESTIGATION

3.1 STRENGTH AND DETERMINATION COMPARISON

According to the aim of this research, the shear carrying capacity of reinforced concrete member with a small amount of web reinforcement is the most important in consideration and outcome. Thus, shear capacity of beams are now determined and compared with the experimental result as it shown in Table.3_1.

Table.3_1: Comparison table.

Categories	Beam 1	Beam 2	Beam 3	Beam 4
1.) Compressive strength (Mpa)	41.5	43.5	41.7	46.15
2.) Stirrup (%)	0.079%	0.063%	0.048%	0.035%
3.) V_s (kN)	40.0	32.0	24.6	17.8
4.) Shear crack, V_{cr}, (kN)	170.2	157.7	145.6	156.3
5.) Shear crack, V_c, (kN)	160.81	163.35	161.06	166.6
6.) Shear capacity, V_u, (kN)	226.6	**187.8**	**190.8**	**187.5**
7.) V_{cr} / V_c	1.06	0.97	0.9	0.94
8.) V_{cr} + V_s	210.2	189.7	170.2	174.1
9.) V_c + V_s	200.81	195.35	185.66	184.4
10.) V_u / V_{cr}+V_s	1.08	0.99	1.12	1.08
11.) **V_u / V_c+V_s**	1.12	0.96	1.03	1.02
12.) Failure mode	S.C.*	S.C.	S.**	S.C.

* Shear compression failure
** Shear failure

Refer to the table of comparison, the category number 11 and 12 show that the design concept by superposition method, no matter what shear crack strength is taken from either experiment or calculation, is good enough to predict the shear carrying capacity of reinforced concrete members with a small amount of web reinforcement. Even though the shear carrying capacity of each beam, except Beam 1, has mostly the same level, this is going to quote and investigate into the mechanisms in next chapter.

3.2 DISCUSSION ON EXPERIMENTAL RESULTS

3.2.1 General Behaviors

Beam 1: *The strongest beam*
This beam contained the highest amount of web reinforcement, about 0.08%, which is less than the design code mentioned in the first chapter. As observed during the test, shear crack was taken placed at the shear load level equal to 170.2 kN. This beam did not illustrate large and clear dropping of load deflection curve when the shear crack was taken place. It can be seen from the load deflection curve, Figure.3_1 that the shear crack load level did not affect in any drop of plotting.

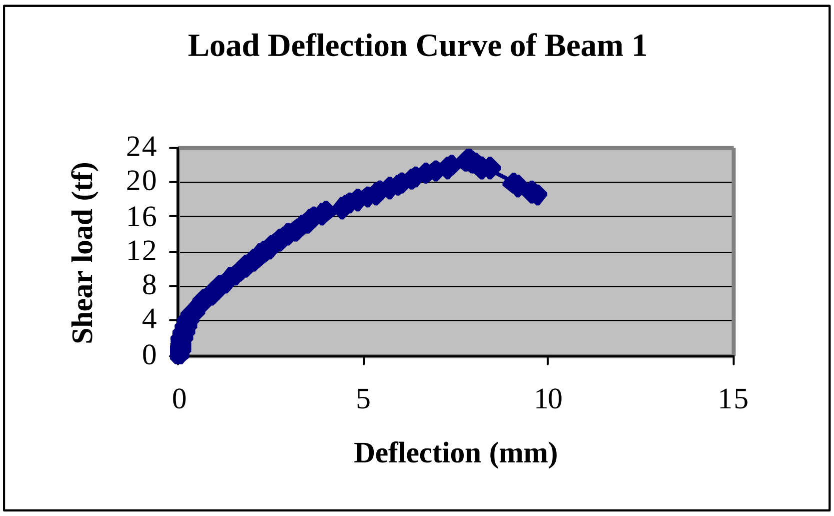

Figure.3_1: Load deflection curve of Beam 1.

After shear cracking formed, the load was increased further up to the failure point and the beam was fail as shown in Figure.3_2. Regarding to failure load, the compression zone was compression failure and followed by the large opening of shear crack since the shear resisting in compression zone suddenly dropped and aggregate interlocking at shear crack surface can not substitute shear resisting in compression zone. This phenomenon simultaneously taken place with the cut-off of web reinforcement illustrated in Figure.3_3.

Figure.3_2: Shear failure of Beam 1.

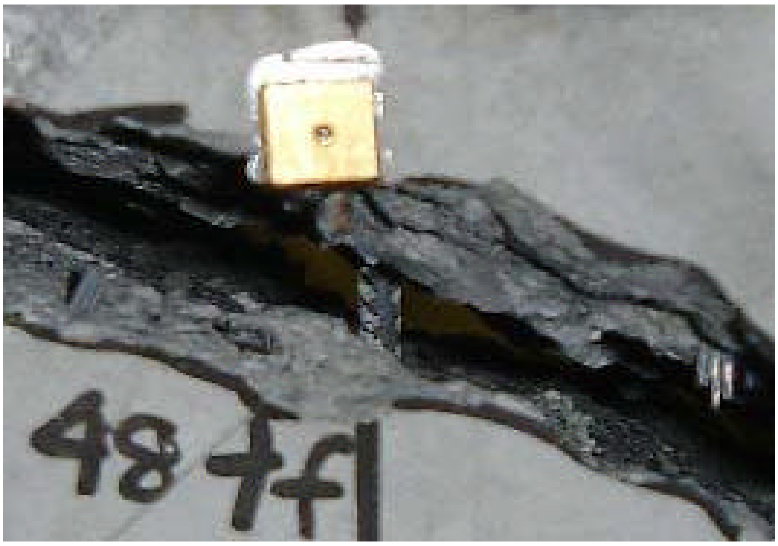

Figure.3_3: Cut-off of Stirrup

Beam 2: A second highest amount of web reinforcement

Shear crack was taken place at load level in shear span equal to 157.7 kN, which is slightly less than that of Beam 1. The crack formation of Beam 2 was generally similar to Beam 1 without any special consideration. Shear crack formation, shear failure, and load deflection curve are shown in Figure.3_4

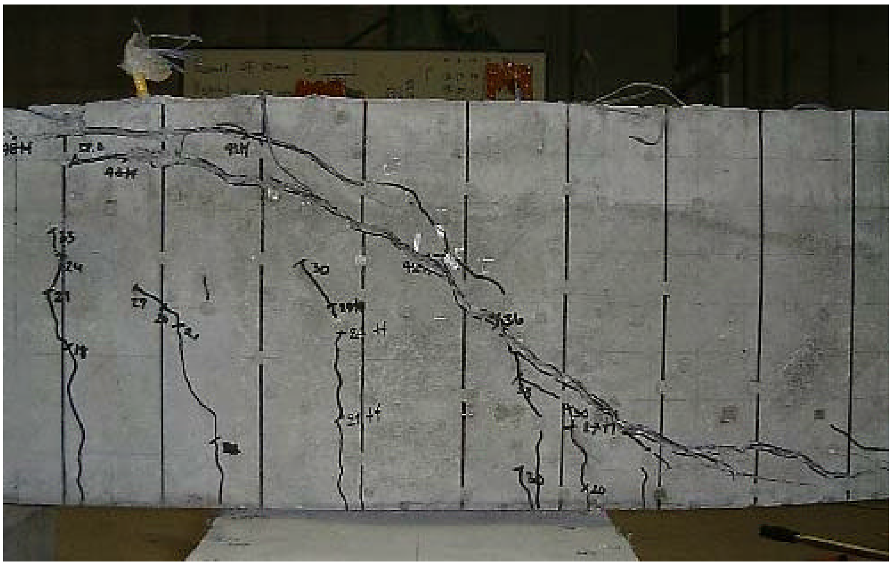

Figure.3_4: Shear failure of Beam 2.

It is possible that the larger amount of web reinforcement ought to delay the time to generate the perfect shear crack since the web reinforcement has highly possibility to intersect with crack and effectively activate at the same load level as shown in Figure.3_5 *a* and *b*. Similarly, it is too difficult to judge when the shear crack became complete path since it does not occur suddenly as the case of beam without web reinforcement. Therefore, it has possibility that the excess shear crack strength in last beam compare to calculation is routed by them.

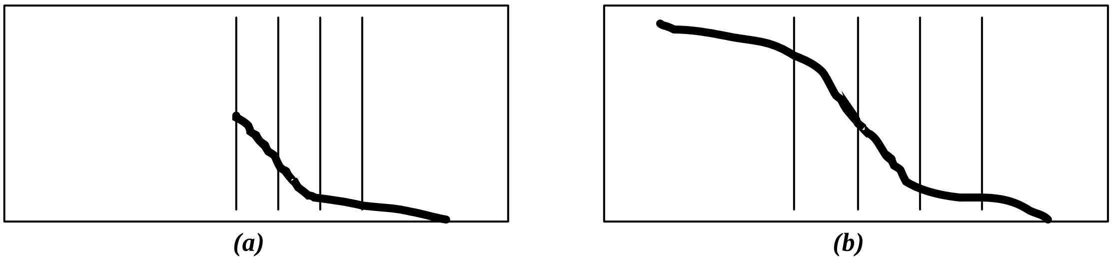

Figure.3_5: Possibility of crack intersect with stirrup

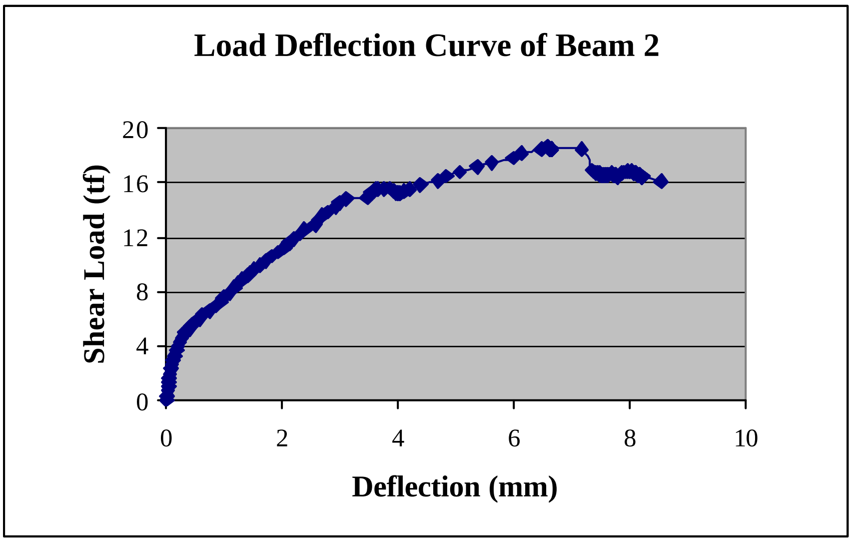

Figure.3_6: Load Deflection Curve of Beam 2.

Onset of shear crack started, as it can be observed, Figure.3_6, load deflection curve shows a little drop of graph after shear crack. It meant that the shear crack penetrated to the compression quite fast, and then the load was able to increase further up to the failure point. Similar to previous case, the compression zone firstly came up to the compression failure, and then followed by the large shear crack opening and cut off of web reinforcements in simultaneously.

Beam 3:

By the same manner with two previous beams but it is important to mention, shear crack load of Beam 3 was observed at the shear load level equal to 145.6 kN that is quite low when compares to previous cases. Even though this beam possessed the same compressive strength and geometry as Beam 1, the shear crack loads of these two beams were observed at different level of shear load. With respect to this phenomenon, it is believed to be the same reason as explained in Beam 2 but it seems to be clear since both beam had the same compressive strength. Shear crack formation, shear failure, and load deflection curve are also shown in Figure.3_7.

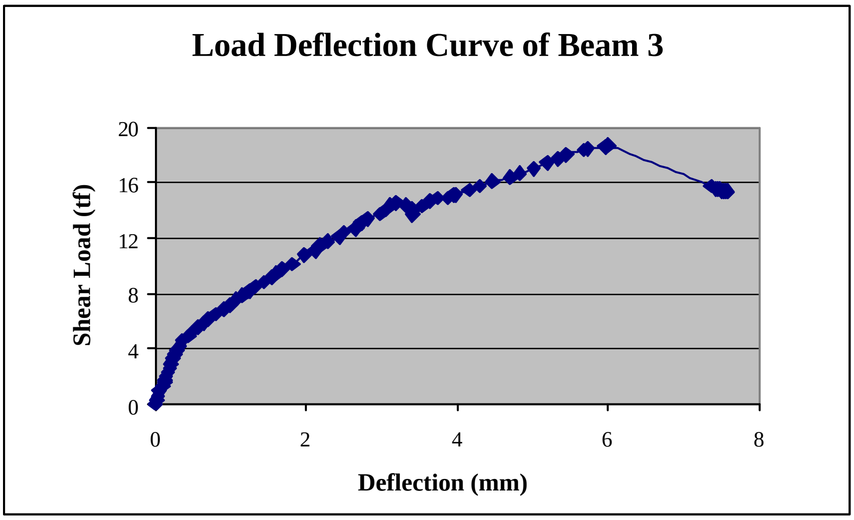

Figure.3_7: Load Deflection Curve of Beam 3.

Figure.3_8: Shear Failure of Beam 3.

From the load deflection curve, the load had a little drop as shear crack taken place and then the load was increased until the failure occurred. Different mode of failure was observed in this beam since the compression zone did not fail, but the shear crack was the controller instead.

Beam 4: Minimum web reinforcement

The smallest amount of web reinforcement beam, Beam 4, was created shear crack at shear load level equal to 156.3 kN. Although the compressive strength of cylindrical specimen was the highest in all beams, the shear crack load level was not the highest among them. At the first place, this is believed that the initial state of crack did not confine by the web reinforcement so that the stress at the crack front was propagate very fast as aforementioned.

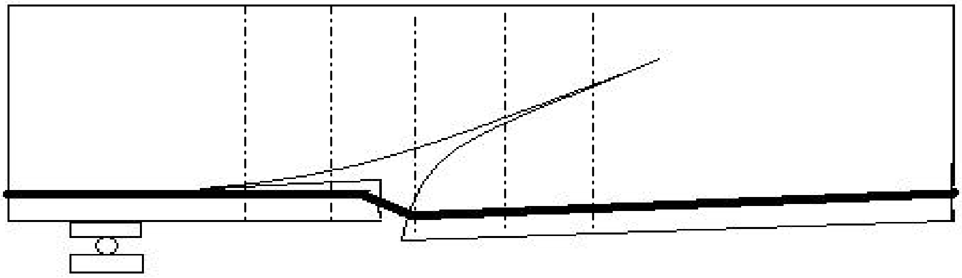

Figure.3_9:Geometry of crack propagation.

In more details to support previously claimed, it was discovered that bond between the concrete and the main reinforcement is a major cause for the initiation of inclined shear cracking in RC beams [1]. Since this bond are shown to have excellent high shear stress induce at the main reinforcement height. Its propagation believed to be controlled by longitudinal cracking along the main bar, so-called horizontal crack [2], which actually comes after inclined shear cracking. Therefore, the less amount of web reinforcement usually yields lower load level to resist horizontal crack will show larger and faster horizontal crack, which finally results in faster perfect shear cracking. The illustrative sketching can be shown as Figure.3_9. However, this is not dowel action developing by web reinforcement since the dowel action is believed and shown to be affected by the position of first stirrup from the crack [3].

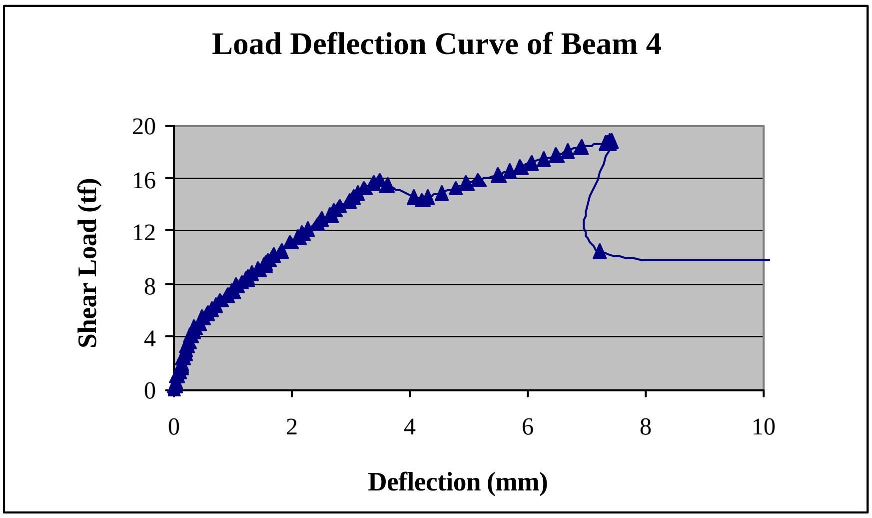

Figure.3_10: Load Deflection Curve of Beam 4.

As shown in Figure.3_10 and 3_11, they illustrate about load deflection and shear failure pattern. According to the load deflection curve, there is very large drop of it since it was not enough amount of stirrup to confine the crack in both incline shear crack and horizontal crack [3]. However, onset of shear cracking, the equilibrium became satisfy again, and then the shear load can be increased further up to the failure point. The failure mechanism and pattern in this beam is similar to Beam 1 and Beam 2, which the compression zone firstly crushed, and followed up by large crack opening and cut-off of stirrups.

Figure.3_11: Shear Failure of Beam 4.

3.2.2 Shear Carrying Comparison among Beams

Beams behaviors under experiment were explained as aforementioned; currently, shear carrying capacity of all beam are going to be compared. Since the compressive strength in each beam is different; therefore, the comparisons have to be done by normalizing compressive strength. However, dividing the shear load by compressive strength seems no meaning. Fortunately, there are some equations for determining shear crack load and those equations used the compressive strength to be one of parameter [4,5].

By normalizing the effect of compressive strength term, the remaining terms are concerned with the beam geometries, and all beams had used the same geometry during this experiment. Therefore, the comparing result by normalizing the compressive strength term in shear crack prediction formula is used, and then it consequently shows in Figure.3_12.

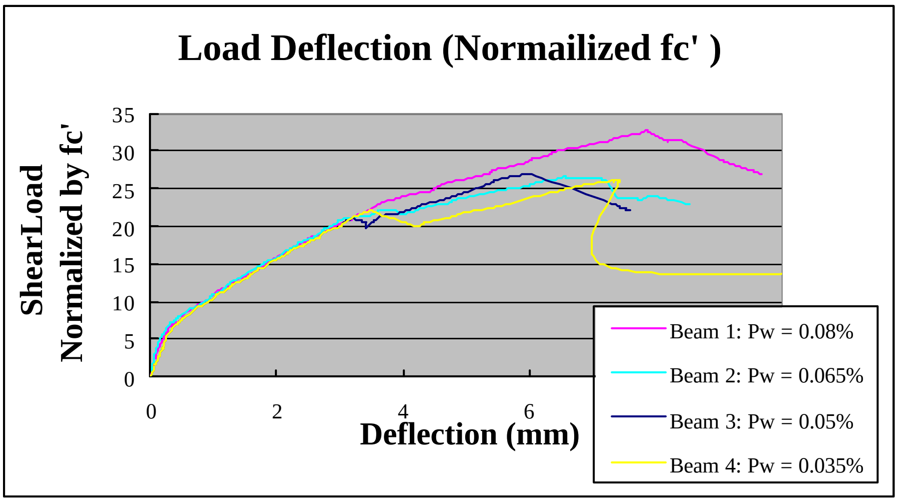

Figure.3_12: Shear carrying capacity comparison by normalizing compressive strength.

As observed from the graph, the effect of compressive strength on the shear crack load was eliminated but the shear crack load and shear carrying capacity are still the same, except Beam 1. It is meant that at a small amount of web reinforcement shear carrying capacity cannot be exactly calculated by the concept of superposition; however, the superposition method is still good enough to safely determine the shear carrying capacity. As claimed, in actually, the bending members without web reinforcement of the shear span ration, a/d, equal to 3.0 has some reserve strength after the shear crack was taken place. It can be simply said that the shear strength of the short span beam is not shear crack load.

3.2.3 Comparison between Several Calculation of Shear Strength

Several design codes, such as Eq.3-1, 3-2, 3-3, and 3-4, used superposition method by selecting the concrete contribution term equal to the shear crack load. Currently, it should be know that "Can these codes satisfy the shear failure beams with a small amount of web reinforcement?" The calculation results are shown in Table.3_2.

ACI Committee 318-95 [6]

$$V_{cr} = \left(0.16\sqrt{f_c^{'}} + 17.2\, r_s \frac{Vd}{M}\right) b.d \qquad 3\text{-}1$$

CSA Simplification [7]

$$V_{cr} = \left(\frac{220}{1000+d}\right)\sqrt{f_c^{'}}\, b.d \qquad 3\text{-}2$$

CEB-FIP [8]

$$V_{cr} = 150.\left(1+\sqrt{\frac{0.2}{d}}\right)\left(\frac{3d}{a_s}\right)^{1/3}(100.\,r_s)^{1/3}\, f_c^{'1/3}\, b.d \qquad 3\text{-}3$$

JSCE 1986 [9]

$$V_{cr} = 0.9\, f_c^{'1/3}(100\, p_s)^{1/3}(100/d)^{1/4}\, b.d \qquad 3\text{-}4$$

Table.3_2: Comparison between Experiment Results and Calculation of Several Codes.

Specimens. No.	ACI 318-83	CSA 1994	CEB-FIP	Newzealand NZS 3101	JSCE 1986
Beam 1	1.15	1.19	1.42	1.10	1.17
Beam 2	0.98	1.01	1.22	0.93	1.19
Beam 3	1.05	1.09	1.32	1.00	1.19
Beam 4	1.03	1.07	1.32	0.97	1.21

Note: The values in table obtained from dividing the experimental with calculation.

Mostly, the design code of each country used the shear crack load in estimating shear carrying capacity in superposition method. Therefore, the superposition method is seemingly applicable and good enough. With respect to safety, the comparisons table shows that the testing values to calculating values ratio nearly to 1.0 and underestimation in some codes, such as CEB-FIP and JSCE, since all of them were provided some safety factor in the derivation processes. Especially, CEB-FIP shear strength model was derived from fracture mechanic by assuming the dynamic mode of splitting crack. This mode of splitting crack in the horizontal cracking is actually not dynamic but the quasi-statistic by considering with the energy used in opening conical crack at the bond face [10].

However, it has to be reminded again that the tested beam had quite short span ratio but it is scale down in dimension from Shinkansen viaduct structures. As it is short, the shear strength of the beam is not exactly the shear crack load but larger than that.

Implying of inefficiency stirrup can be highly possible, and it can be said in advance that simply said that the superposition method is not safe for long shear span ratio.

3.3 CONCLUSION

1.) The shear carrying capacity of reinforced concrete beam with a small amount of web reinforcement can be safely determined by the superposition method, Vc+Vs.

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CHAPTER 4

MECHANICAL IN SHEAR RESISTANCE

- Mechanism of Stress Transfer
- Mechanism of Web Reinforcement

MECHANICAL IN SHEAR RESISTANCE

In generally, the shear resistance can be divided into concrete part and web reinforcement part. According to the contribution of concrete part, the aggregate interlocking, dowel action, and shear in compression zone will be raised up after shear cracking. However, in this research, the measured component was only the aggregate interlocking or shear crack deformation since there is a limitation in techniques and accuracy. Finally, it can be said that the shear resistance mechanism of reinforced concrete beam with a small amount of web reinforcement was obtained.

4.1 MECHANISM OF STRESS TRANSFER

4.1.1 Crack Deformation

Due to the geometry of shear crack, some of contact chip groups are properly selected to use in determining shear stress across the crack. Fortunately, all beams mostly yielded the same crack pattern so that the shear stress across the crack can be directly compared to each other. Before going to compare the shear stress cross the crack, the location of measurement is shown by the circles in Figure 4_1.

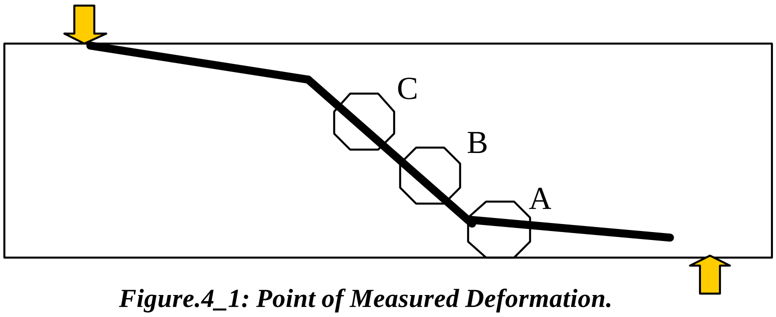


Figure.4_1: Point of Measured Deformation.

At the considered location of crack deformation, the geometry of crack and beam behavior have definitely correlation as Figure 4_2. Onset of shear crack, the crack located in compression zone and the horizontal cracking, tension bars, mainly behave in opening and rarely exist in sliding since the crack angle is almost parallel to the beam axis. In other hand, as the horizontal crack opening, the inclined crack is going under opening and sliding, simultaneously.

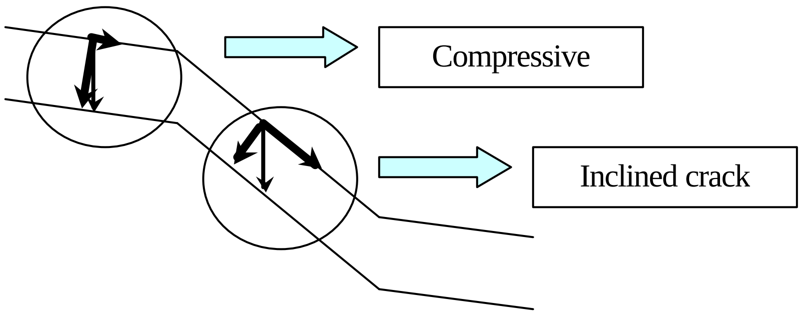


Figure.4_2: Crack Geometry and its deformation.

The magnitude of crack opening and sliding can be considered by the length of the arrow vector. By this figure, the above-mentioned are brightly understood the total different at the compression zone and the inclined crack location.

Point A: Horizontal crack

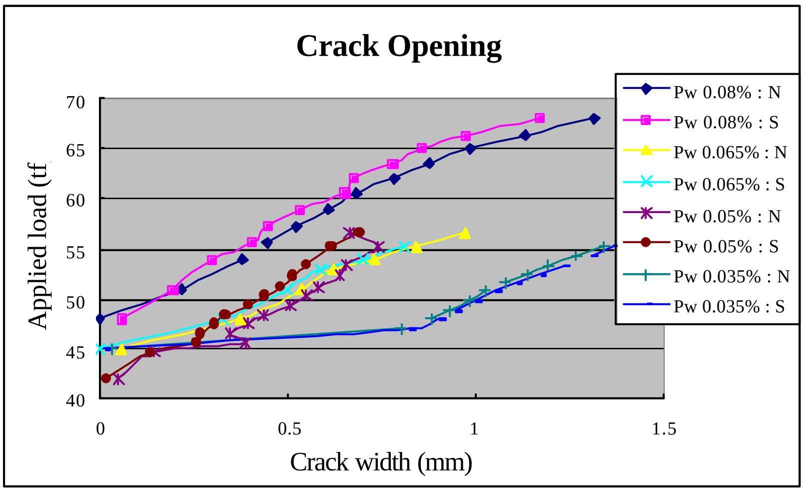


Figure.4_3(a): Crack opening at the horizontal cracking.

In Figure.4_3 (a), beam specimen with web reinforcement ratio equal to 0.035% shown the highest crack opening, whereas other beams with 0.065%, 0.05%, and 0.08% web reinforcement show smaller crack width at the same load level, respectively. Even though the beam with web reinforcement ratio equals to 0.05% should have crack width value larger than that of 0.065% but it is inversely perceived from the measuring. However, beam with 0.05% web reinforcement ratio shown the different in failure mode among others. Accordingly, it is believed that the reason routed from different in failure mode. It can be concluded that the crack width of beam at horizontal cracking will become larger respect to the smaller amount of web reinforcement used.

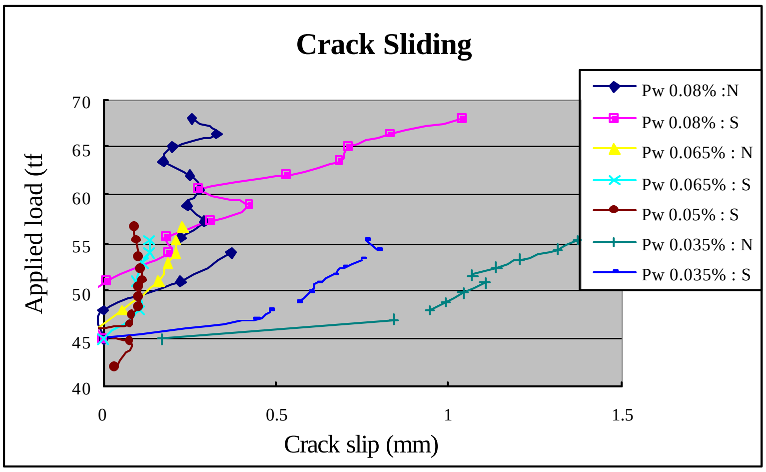


Figure.4_3(b): Crack sliding at the horizontal cracking.

Definitely different from the crack opening at the horizontal cracking, the crack sliding shown in Figure.4_3 (b) does not clearly show the good relation between crack sliding with amount of stirrup used. However, it can be observed a little bit larger in crack sliding in beam with 0.035% web reinforcement and almost same magnitude of crack sliding in beam with 0.05%, 0.065%, and 0.08%, of web reinforcement.

Point B: Bottom of inclined crack

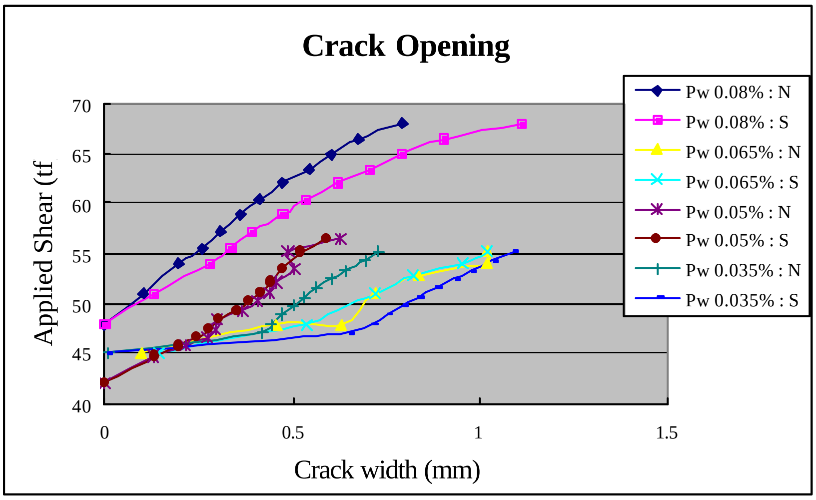


Figure.4_4(a): Crack opening at the bottom of shear cracking.

At the bottom of shear cracking, crack opening is largest when the web reinforcement is smallest, and it became smaller when larger amount of web reinforcement is used as observed in Figure.4_4(a). However, the beam with web reinforcement 0.05% should have larger crack width than that of 0.065% but it is not actually since the failure mode was different in beam that used 0.05%. These are also evidenced in crack sliding as it shown in Figure.4_4(b).

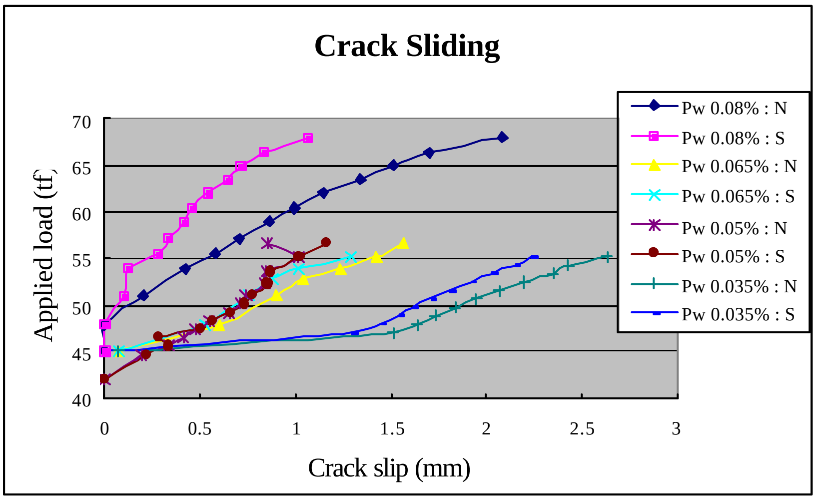


Figure.4_4(b): Crack sliding at the bottom of shear cracking.

Furthermore, it can be observed that beam with web reinforcement 0.035% has very large crack opening and sliding suddenly onset of shear crack initiated but it was not the case for other beams.

Point C: Top of inclined crack

At the last location, tip of shear crack, the crack deformation, opening and sliding, is shown in Figure.4_5 *(a)* and *(b)*. With respect to the measuring result in figures, the same behavior with last two locations can be observed.

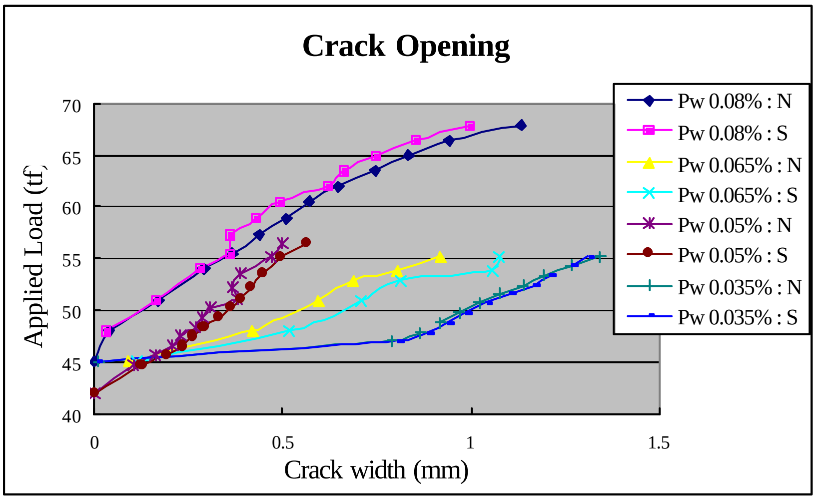


Figure.4_5(a): Crack opening at top of shear cracking.

More deeply in detail of crack deformation, it can be observed that the crack width was almost the same in three reference locations but it was highest in crack sliding at the bottom of shear crack width. Finally, again, it is absolutely possible to said that when the larger amount of web reinforcement were used, the smaller crack deformation in both opening and sliding without regarding to the location of total shear cracking path.

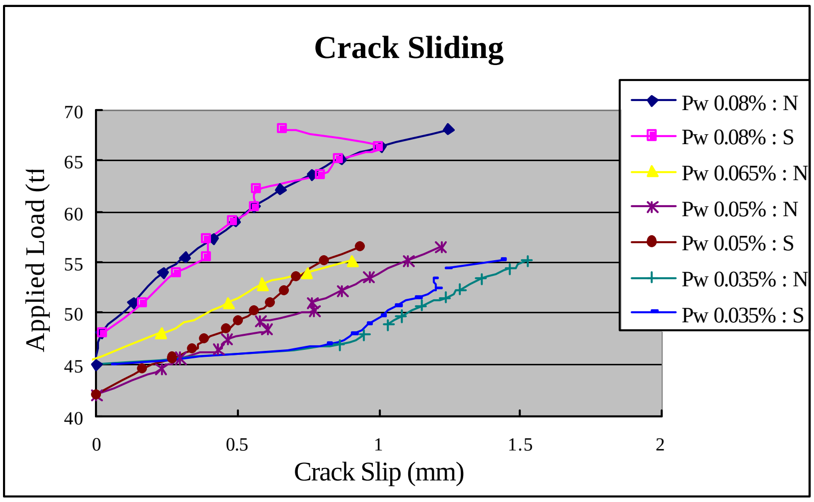


Figure.4_5(b): Crack sliding at top of shear cracking.

4.1.2 Crack Deformation Path

From Fig.4_6, the plots of crack sliding to opening ratio at the lower side of inclined crack are illustrated. In this figure, the applied shear load of all beams was normalized by the maximum applied shear load of beam with highest shear carrying capacity, which was the beam with web reinforcement ratio 0.08%. Except the beam with web reinforcement ratio

0.05% that failed in different manner compare with others, it can be observed that the plots of crack sliding to opening ratios were approximately regarded to be the same level.

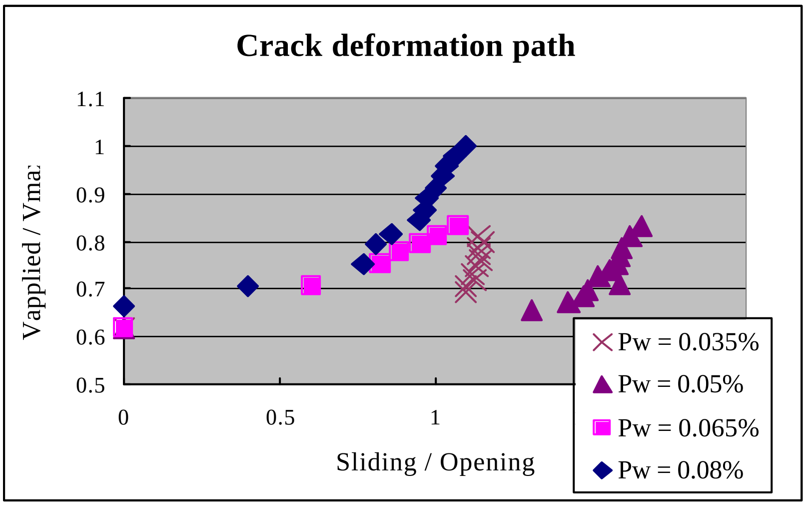


Fig. 4_6: Ratio of crack sliding to opening.

However, it can be also observed from this figure that the beams with smaller web reinforcement had a large amount of crack sliding and crack opening just after the shear cracking started. On the other hands, the beam with larger amount of web reinforcement behaved only crack opening in the first state, and then gradually increased its sliding.

4.1.3 Shear Stress Cross Crack (Aggregate Interlocking)

By summing up all measured point, the aggregate interlocking contributed to resist the shear load in each beam is calculated. With respect to results, the characteristic in stress transfer across the crack of each beam can be easily seen from the graph as shown in Figure.4_7. Start with Beam 2, 3, and 4, have similarly shear transfer, which they are 100 kN up to 140 kN. Scattering is evidently observed but it is the limitation of method used to measure the crack deformation. Then, last specimen, Beam 1, has a great different stress transfer across the crack from others, where it starts with small value after the crack has initially propagated until a little bit higher load than the perfect shear crack taken place. After that, the shear stress across crack gradually rise up to the 160 kN level, which was the highest among 4 specimens, and then the shear failure is consequence.

Related to the beams behavior during the experiments, the three smallest in web reinforcement specimens evidently show the dropping of load resistance since the shear crack was suddenly opening and propagate as load deflection curves observed in previous chapter. Thus, the suddenly jump to the maximum or near maximum of stress transfer across the crack is the result of changing equilibrium, jump. Nevertheless, it was not the matter for Beam 1 since the crack was not suddenly open and propagate so fast that the load shown evidently dropping. Therefore, the shear transfer did not rapidly increase but it delayed until the crack was widely open and went under sliding. With respect to these considerations, the graph of stress transfer across crack was shown in somewhat different patterns; however, the maximum shear transfer can approximately the same with some degree of accuracy.

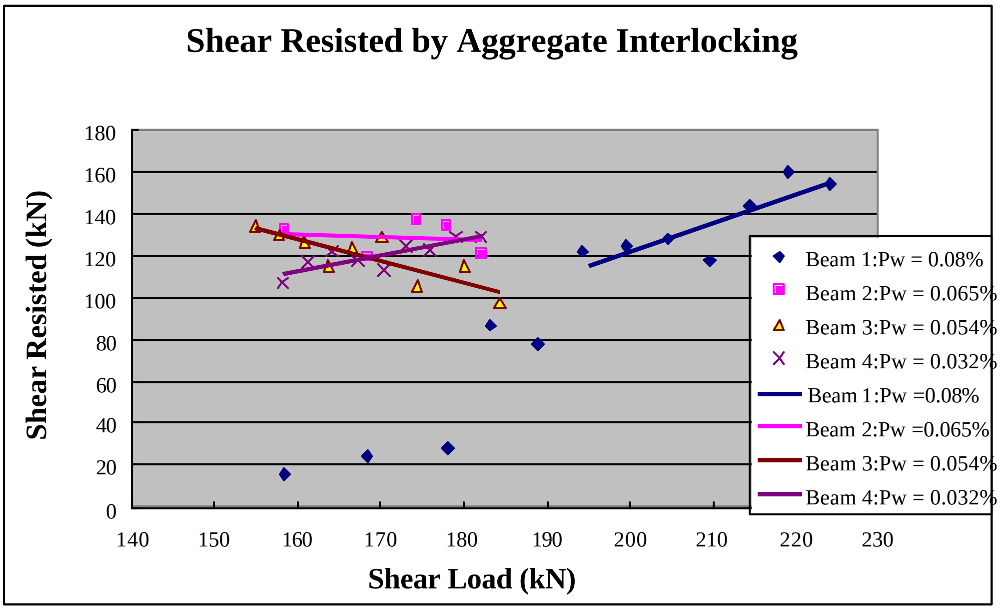


Figure.4_7: Stress transfer cross crack.

Finally, the most important item is the crack opening and sliding. Due to these, in reinforced concrete beam with a small amount of web reinforcement, it is possibly that the beam with larger crack width corresponding to the larger sliding wins to the same level of aggregate interlocking or stress across crack as beam with smaller crack width and sliding since it can be previously observed in shear transfer of beam number 2,3, and 4 that used 0.065%, 0.05%, and 0.035% of web reinforcement, respectively. However, beam with web reinforcement 0.08% is approximately respected to have same shear transfer of beams even the first state yielded the low shear transfer and then became large shear transfer.

4.2 MECHANISM OF WEB REINFORCEMENT

4.2.1 Yielding of Web Reinforcement

After the data was collected and carefully considered, it can be said that the collected data from the strain gauges mounted along the web reinforcement cannot be employed. Thus, it seems that the yielding of web reinforcement cannot be known from the direct measurement. However, it is known that crack width equal to the amount of pull out of steel bar, which is 2 times of slip. By considering this relation, it should suit to approximate the slip from the crack width as Eq.4-1.

$$2*Slip = Crack\ Width \qquad 4\text{-}1$$

Not only slip is calculated, but also strain at the crack should be found since the objective is “Did the web reinforcement yield at the shear crack intersection?” With respect to this reason, long embedment length terminology is considered to employ in this case since the web reinforcement is a tiny size compared with the height of the beams. Therefore, 2 more equations of strain-slip relation [1] for long embedded bar are tool for the analysis in this state, and those equation are specified in following;

$$s = e_s\left(6 + 3500e_s\right) \qquad 4\text{-}2$$

$$s = \frac{Slip}{D}\left(\frac{f_c^{'}}{20}\right)^{2/3} \qquad 4\text{-}3$$

Where,

s	:	normalized slip
e_s	:	strain of web reinforcement
D	:	diameter of web reinforcement
$f_c^{'}$	:	cylindrical compressive strength of concrete

At this state, after all related equations was introduced, the yielding strain from the material test are going to be input in determining the normalized slip, and then slip and crack width can be finally determined. Follow that describe, it can be finally calculate that the crack width corresponding to the strain at yielding, which determines by 0.002 offsets, is 0.42 mm, and then compare to the crack width of every measured location along the shear crack, for example Figure.4_3, 4_4, and 4_5. In conclusion, all web reinforcement intersected with the shear crack can be considered as yielding state before shear failure of the beams according to the previous figures of crack opening and sliding.

4.2.2 Other Mechanisms Contributed by Web Reinforcement

As frequently referred to the strain of web reinforcement, the effect of unachievable in this value influences on the analysis of other mechanisms such as shear resistance in compression zone and dowel action, which they cannot direct measure except adopt from the indirect calculation. Therefore, it has to state in here that those mechanisms [2] have not been investigated for reinforced concrete beam with a small amount of web reinforcement currently.

4.3 CONCLUSION

1.) The beam with smaller amount of web reinforcement show larger in crack opening and sliding. Except beam with 0.05% of web reinforcement, the crack opening and sliding show some different from the claimed relation since the failure is absolutely different.
2.) Beams with a small amount of web reinforcement have the same stress transfer across the crack even the crack opening of them are totally different. Since the larger of crack opening shows larger of crack sliding, the stresses in those beams are finally maintained to be the same.
3.) All web reinforcement intersected by shear crack is considered to reach yielding state by relying on the strain-slip model of long embedded steel bar and the relation of crack width and slip.

4.4 REFERENCES

1.) Okamura, H., and Maekawa, K., "Nonlinear Analysis and Constitutive Models of Reinforced Concrete," Gihodo, Tokyo, 1991.
2.) Park, R., and Paulay, T., "Reinforced Concrete Structures," John Wiley & Sons, New York, 1975.

CHAPTER 5

CONCLUSION

CONCLUSION

Result of this research is very useful even the applicable length is very specific since the experimental work was done only in the range of shear span equal to 3.0. However, it can be said that most of viaducts and buildings are composed with the columns of shear span ratio 3.0. According to the experimental results of four tested beams and their corresponding measurement values of aggregate interlocking; therefore, it can be confidently concluded as a unit as following:

1.) The shear carrying capacity of reinforced concrete beam with a small amount of web reinforcement as shear span ratio 3.0 can be safely determined by the superposition method, $V_{cr}+V_s$.

2.) The beams with smaller amount of web reinforcement showed larger in crack opening and sliding. Except beam with 0.05% of web reinforcement, the crack opening and sliding showed some different from the claimed relation since the failure is absolutely different.

3.) All beams have the same stress transfer across the crack even the crack opening and sliding of them were totally different. Since the larger of crack opening showed the larger of crack sliding; therefore, the stresses in those beams are finally maintained to be the same.

CHAPTER 6

RECOMMENDATIONS OF FURTHER STUDY

RECOMMENDATIONS OF FURTHER STUDY

Based on the experimental results according to the experimental plan and problems in analyzing procedures, it is better to specify some recommendations for further study as following:

1.) Improving on measuring systems or data collecting method, for example stress transfer across crack and strain of web reinforcement at the crack position, in order to improve the accuracy and more detail understanding in the depth of behavior.

2.) Extending the measuring systems or data collecting method to other locations or items, for example the shear resistance of compression zone and that of dowel action.

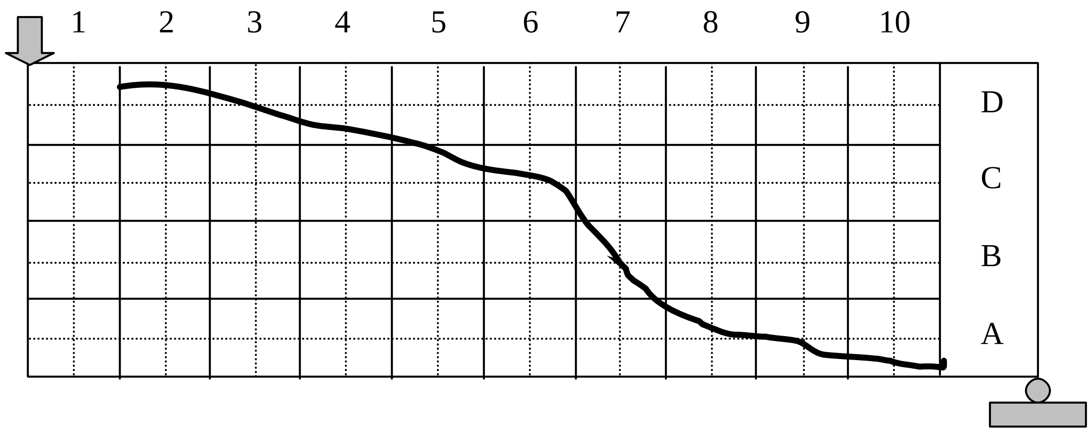
3.) More widely parametric studies are necessary to continue since this behavior seemingly to different in longer shear span ratio, compressive strength, and absolutely different in reversed cyclic loading.

APPENDIX A

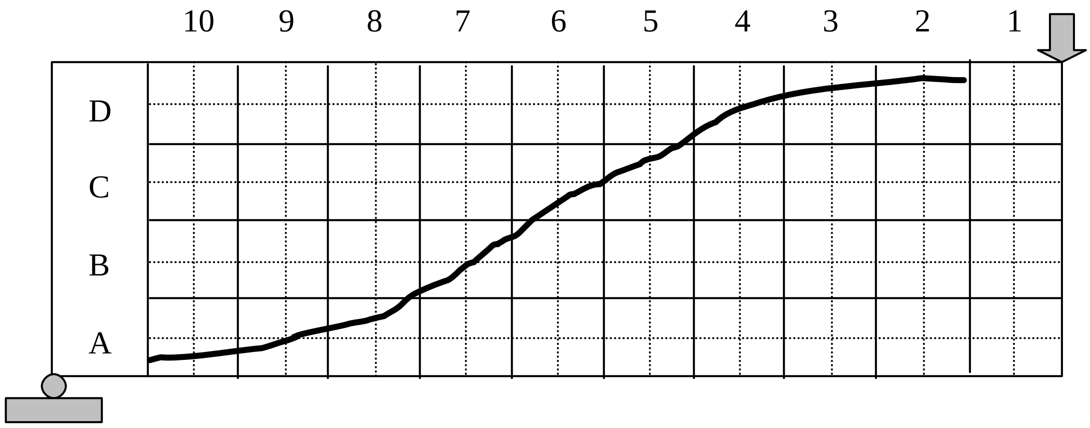
Crack Mapping

BEAM WITH WEB REINFORCEMENT 0.08%

North side

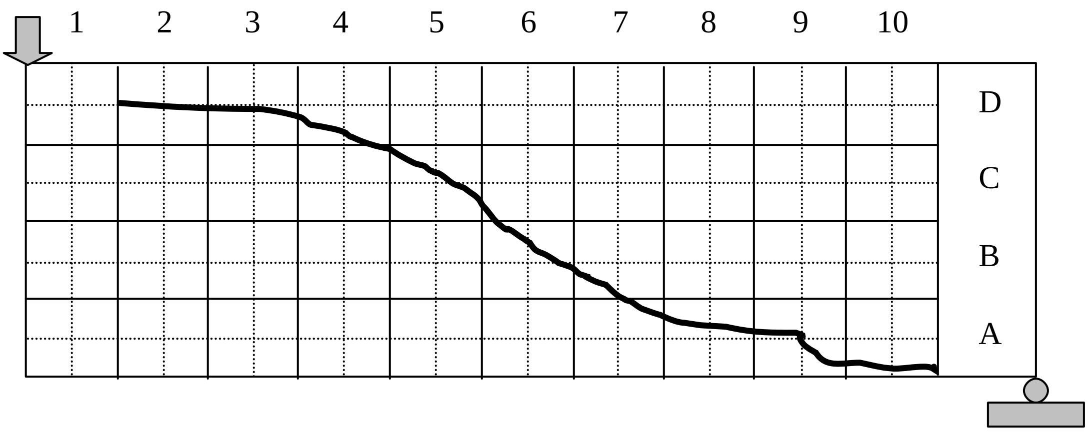


South side

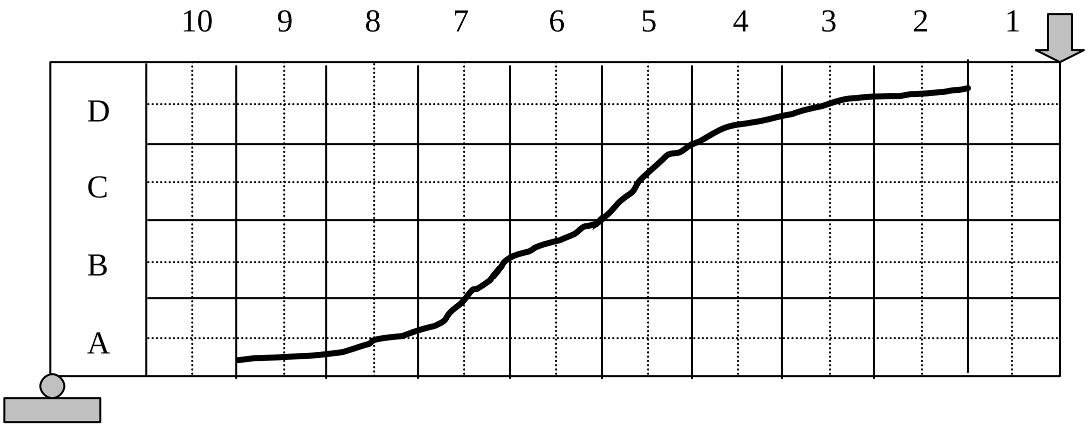


BEAM WITH WEB REINFORCEMENT 0.065%

North side

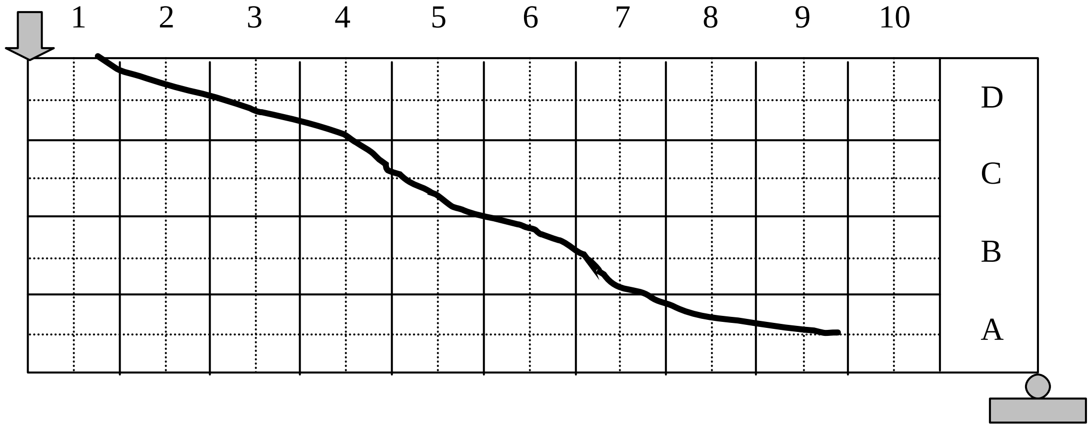


South side

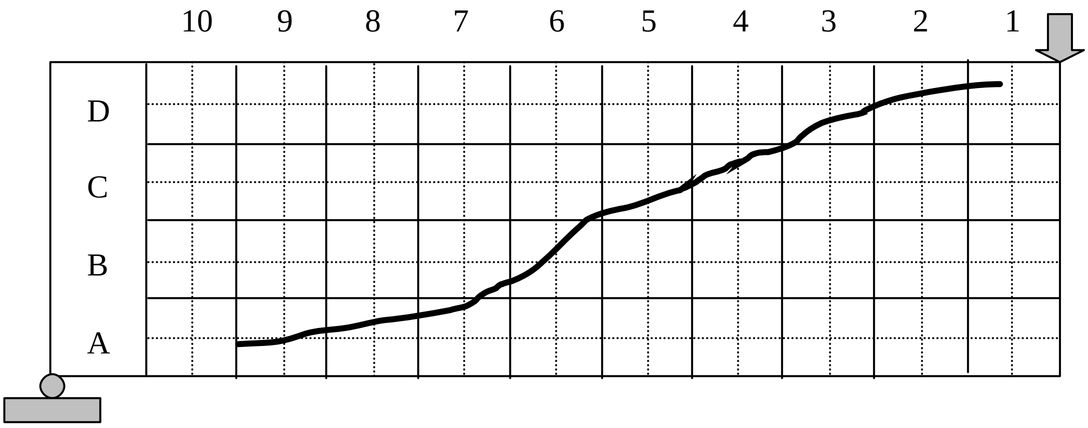


BEAM WITH WEB REINFORCEMENT 0.05%

North side

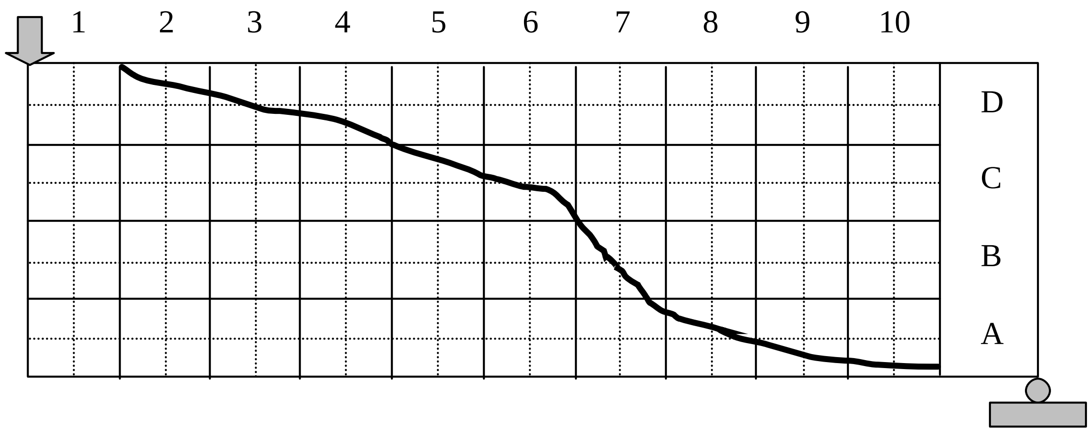


South side

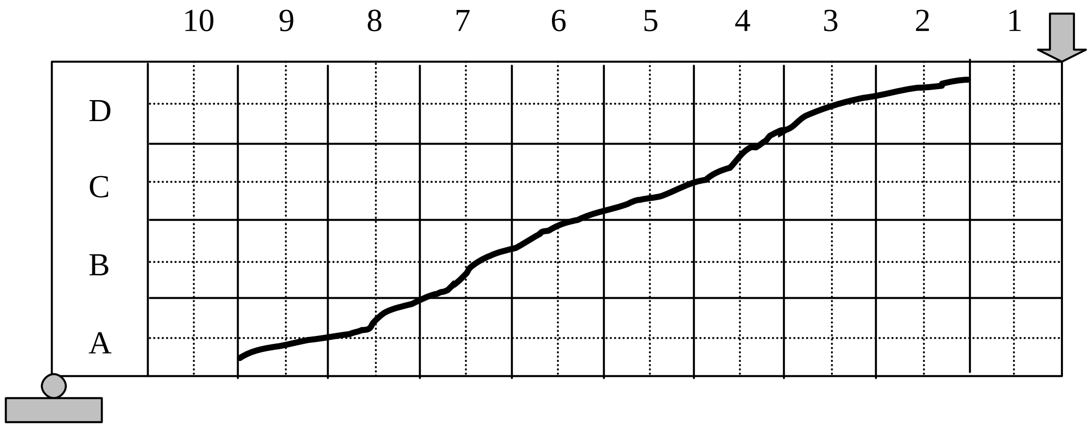


BEAM WITH WEB REINFORCEMENT 0.035%

North side



South side



APPENDIX B

Crack Deformation at Shear Cracking of Each Beam

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number North (A9)
Crack Angle 10 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	0.001	0.002	-0.017	0.0018	-0.0011	-0.0176
45	0.008	0.003	-0.001	0.0025	0.0002	-0.0053
51	0.101	0.152	0.242	0.0158	0.0475	0.0998
54	0.083	0.181	0.23	0.0131	0.0529	0.0753
55.5	0.043	0.19	0.196	0.0051	0.0532	0.0496
57.24	0.038	0.194	0.178	0.0054	0.0526	0.0316
58.9	0.01	0.217	0.13	0.0026	0.0541	-0.0199
60.47	0.015	0.185	0.178	-0.0016	0.0516	0.0442
62.04	0.172	0.252	0.183	0.0461	0.0599	-0.0409
63.5	-0.126	0.26	0.17	-0.0374	0.0709	0.0308
65	0.006	0.25	0.217	-0.0043	0.0683	0.0419
66.4	0.003	0.324	0.102	0.0084	0.0733	-0.1127
67.95	0.029	0.578	0.415	0.0019	0.1499	0.0109

Element Number North (A9)
Crack Angle 25 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	0.001	-0.008	-0.017	0.0024	-0.0042	-0.0052
45	0.001	0.003	-0.001	0.0009	0.0001	-0.0027
51	0.115	0.152	0.242	0.0096	0.0571	0.0556
54	0.083	0.181	0.23	0.0064	0.0596	0.0255
55.5	0.043	0.19	0.196	0.0021	0.0562	-0.0052
57.24	0.038	0.194	0.178	0.0046	0.0534	-0.0199
58.9	-0.012	0.217	0.13	0.0020	0.0493	-0.0700
60.47	0.037	0.185	0.178	0.0030	0.0525	-0.0136
62.04	0.03	0.252	0.183	0.0094	0.0611	-0.0580
63.5	0.016	0.26	0.17	0.0088	0.0602	-0.0729
65	0.006	0.25	0.217	-0.0046	0.0686	-0.0362
66.4	0.003	0.324	0.102	0.0269	0.0549	-0.1625
67.95	0.029	0.578	0.415	0.0104	0.1413	-0.1386

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number North (A8)
Crack Angle 20 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.064	0.034	0.104	0.0063	0.0182	0.0518
45	0.105	0.047	0.1	0.0207	0.0173	0.0370
48	0.039	0.115	0.153	-0.0002	0.0387	0.0338
51	0.021	0.426	0.486	-0.1032	0.2149	0.2216
54	0	0.349	0.351	-0.0746	0.1618	0.1460
55.5	0.033	0.337	0.127	0.0265	0.0660	-0.1421
57.24	0.005	0.228	0.294	-0.0208	0.0790	0.0643
58.9	-0.001	0.336	0.244	-0.0027	0.0864	-0.0497
60.47	0.04	0.239	0.27	-0.0052	0.0749	0.0360
62.04	0.006	0.366	0.306	-0.0073	0.1003	-0.0238
63.5	0.027	0.381	0.249	0.0099	0.0921	-0.0793
65	-0.015	0.359	0.365	-0.0238	0.1098	0.0276
66.4	0.036	0.414	0.548	-0.0319	0.1444	0.1259
67.95	0.026	0.698	0.548	-0.0037	0.1847	-0.0735

Element Number North (B7)
Crack Angle 50 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.027	0.004	0.024	0.0013	0.0065	0.0098
45	0.006	0.093	0.049	0.0144	0.0104	-0.0428
48	0.044	0.145	0.191	0.0021	0.0452	-0.0665
51	0.163	0.534	0.442	0.0722	0.1021	-0.1989
54	0.07	0.474	0.423	0.0396	0.0964	-0.2252
55.5	0.053	0.338	0.275	0.0355	0.0623	-0.1541
57.24	0.034	0.256	0.222	0.0221	0.0504	-0.1227
58.9	0.029	0.344	0.237	0.0410	0.0522	-0.1639
60.47	0.041	0.287	0.211	0.0348	0.0472	-0.1293
62.04	0.059	0.331	0.285	0.0325	0.0650	-0.1496
63.5	0.042	0.4	0.32	0.0386	0.0719	-0.1935
65	0.034	0.366	0.256	0.0434	0.0566	-0.1732
66.4	0.063	0.414	0.329	0.0450	0.0743	-0.1885
67.95	0.027	0.778	0.519	0.0882	0.1130	-0.3900

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number North (C6)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a₁	*a₂*	*a₃*	*x*	*y*	*xy*
42	-0.044	-0.065	-0.033	-0.0170	-0.0103	0.0198
45	0.046	0.001	0.028	0.0077	0.0040	0.0217
48	0.063	0.14	0.138	0.0127	0.0381	-0.0151
51	0.137	0.529	0.442	0.0352	0.1313	-0.1152
54	0.073	0.469	0.402	0.0146	0.1209	-0.1060
55.5	0.026	0.303	0.248	0.0057	0.0765	-0.0782
57.24	0.034	0.319	0.22	0.0169	0.0714	-0.1017
58.9	-0.006	0.278	0.225	-0.0030	0.0710	-0.0785
60.47	0.038	0.258	0.208	0.0103	0.0637	-0.0653
62.04	0.013	0.307	0.234	0.0056	0.0744	-0.0903
63.5	0.061	0.423	0.321	0.0208	0.1002	-0.1173
65	0.016	0.349	0.271	0.0057	0.0856	-0.0999
66.4	0.018	0.457	0.34	0.0097	0.1090	-0.1388
67.95	0.053	0.807	0.599	0.0238	0.1912	-0.2420

Element Number North (C5)
Crack Angle 25 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a₁	*a₂*	*a₃*	*x*	*y*	*xy*
42	0.034	0.063	0.086	0.0026	0.0216	0.0130
45	0.047	0.083	0.085	0.0095	0.0230	-0.0009
48	-0.005	0.104	0.126	-0.0110	0.0358	0.0074
51	-0.019	0.458	0.391	-0.0163	0.1260	-0.0725
54	0	0.447	0.44	-0.0215	0.1333	-0.0320
55.5	-0.009	0.316	0.246	-0.0055	0.0822	-0.0650
57.24	-0.006	0.238	0.212	-0.0090	0.0670	-0.0317
58.9	-0.352	0.294	0.227	-0.1082	0.0937	-0.0829
60.47	0.358	0.259	0.204	0.1051	0.0492	-0.0293
62.04	-0.015	0.314	0.262	-0.0106	0.0854	-0.0537
63.5	0.012	0.394	0.304	0.0007	0.1008	-0.0814
65	0.022	0.34	0.248	0.0069	0.0836	-0.0787
66.4	-0.026	0.432	0.323	-0.0090	0.1105	-0.0983
67.95	0.014	0.742	0.549	0.0033	0.1857	-0.1689

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number South (A9)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.049	0.01	0.012	0.0136	0.0011	-0.0081
51	0.086	0.746	0.456	0.0541	0.1539	0.2658
54	0.074	-0.244	0.123	-0.0464	0.0039	-0.2417
55.5	0.008	0.184	0.165	-0.0019	0.0499	0.0417
57.24	-0.039	0.218	0.157	-0.0083	0.0531	0.0775
58.9	0.043	0.169	0.19	0.0004	0.0526	0.0126
60.47	0.012	0.191	0.183	-0.0035	0.0542	0.0368
62.04	0.048	0.298	0.191	0.0237	0.0628	0.0993
63.5	0.019	0.322	0.247	0.0071	0.0781	0.0930
65	0.016	0.303	0.241	0.0043	0.0755	0.0835
66.4	0.026	0.309	0.334	-0.0119	0.0956	0.0393
67.95	0.029	0.878	0.585	0.0318	0.1949	0.3019

Element Number South (A8)
Crack Angle 27 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0	0	0	0.0000	0.0000	0.0000
45	0.049	0.081	0.082	0.0105	0.0220	0.0030
48	0.06	0.136	0.233	-0.0084	0.0574	-0.0486
51	0.064	0.484	0.466	-0.0012	0.1382	0.0570
54	-0.003	0.494	0.275	0.0189	0.1039	0.1837
55.5	0.005	0.307	0.384	-0.0293	0.1073	-0.0119
57.24	0.01	0.24	0.063	0.0269	0.0356	0.1295
58.9	-0.002	0.379	0.256	0.0055	0.0888	0.1144
60.47	0.001	0.205	0.49	-0.0675	0.1190	-0.1450
62.04	-0.004	0.323	-0.037	0.0556	0.0242	0.2478
63.5	-0.001	0.474	0.295	0.0124	0.1059	0.1578
65	-0.013	0.251	0.253	-0.0167	0.0762	0.0280
66.4	-0.002	0.464	0.352	-0.0010	0.1165	0.1174
67.95	-0.024	0.797	0.595	-0.0059	0.1991	0.2095

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number South (A8)
Crack Angle 42 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0	0	0	0.0000	0.0000	0.0000
45	0.049	0.081	0.082	0.0116	0.0209	0.0141
48	0.06	0.136	0.233	-0.0101	0.0591	0.0237
51	0.064	0.484	0.466	0.0153	0.1217	0.1888
54	-0.003	0.494	0.275	0.0475	0.0752	0.2441
55.5	0.005	0.307	0.384	-0.0216	0.0996	0.1263
57.24	0.01	0.24	0.063	0.0437	0.0188	0.1209
58.9	-0.002	0.379	0.256	0.0254	0.0689	0.1824
60.47	0.001	0.205	0.49	-0.0731	0.1246	0.0610
62.04	-0.004	0.323	-0.037	0.0845	-0.0047	0.1831
63.5	-0.001	0.474	0.295	0.0384	0.0799	0.2301
65	-0.013	0.251	0.253	-0.0070	0.0665	0.1173
66.4	-0.002	0.464	0.352	0.0216	0.0939	0.2191
67.95	-0.024	0.797	0.595	0.0341	0.1592	0.3865

Element Number South (B7)
Crack Angle 27 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.017	0.002	0.015	0.0023	0.0025	-0.0093
45	0.027	0.062	0.306	-0.0455	0.0678	-0.1317
48	0.065	0.115	-0.09	0.0563	-0.0113	0.1214
51	0.13	0.509	0.426	0.0313	0.1284	0.0975
54	0.075	0.491	0.553	-0.0143	0.1558	0.0215
55.5	0.042	0.332	0.108	0.0429	0.0506	0.1644
57.24	0.038	0.229	0.191	0.0081	0.0586	0.0470
58.9	0.033	0.324	0.252	0.0091	0.0802	0.0795
60.47	0.036	0.25	0.212	0.0065	0.0650	0.0501
62.04	0.043	0.324	0.261	0.0102	0.0816	0.0731
63.5	0.034	0.376	0.26	0.0159	0.0866	0.1110
65	0.029	0.337	0.282	0.0037	0.0878	0.0723
66.4	0.046	0.459	0.344	0.0153	0.1110	0.1200
67.95	0.048	0.843	0.62	0.0196	0.2031	0.2320

Reinforced concrete beam with web reinforcement ratio 0.08%

Element Number South (C6)
Crack Angle 30 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.029	0.001	-0.0055	0.0099	-0.0024	-0.0019
45	-0.001	0.004	0.009	-0.0016	0.0023	-0.0016
48	0.002	0.11	0.099	-0.0021	0.0301	0.0253
51	-0.018	0.51	0.417	-0.0085	0.1315	0.1431
54	-0.023	0.448	0.39	-0.0147	0.1210	0.1152
55.5	-0.059	0.307	0.229	-0.0146	0.0766	0.1060
58.9	-0.034	0.276	0.208	-0.0080	0.0685	0.0907
60.47	-0.002	0.267	0.204	0.0008	0.0654	0.0807
62.04	-0.014	0.346	0.457	-0.0440	0.1270	0.0104
63.5	-0.03	0.359	0.057	0.0401	0.0422	0.2222
65	0.116	0.337	0.282	0.0308	0.0825	0.0679
66.4	-0.077	0.423	0.321	-0.0200	0.1065	0.1425
67.95	0.025	-0.008	0.666	-0.1382	0.1424	-0.3430

Element Number South (C5)
Crack Angle 35 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.07	-0.039	0.088	-0.0085	0.0162	-0.0760
45	0.032	0.064	0.067	0.0062	0.0178	0.0085
48	0.049	0.206	0.155	0.0187	0.0450	0.0644
51	0.113	0.391	0.346	0.0290	0.0970	0.0985
54	0.094	0.399	0.357	0.0226	0.1006	0.1055
55.5	0.043	0.307	0.213	0.0235	0.0640	0.1110
57.24	0.076	0.251	0.207	0.0232	0.0586	0.0673
58.9	0.011	0.264	0.198	0.0179	0.0450	0.1952
60.47	0.013	0.337	0.196	0.0250	0.0625	0.1450
62.04	-0.028	0.388	0.227	0.0162	0.0738	0.1794
63.5	0.068	0.172	0.262	-0.0078	0.0678	0.0003
65	0.006	0.37	0.254	0.0159	0.0781	0.1485
66.4	-0.118	0.402	0.329	-0.0307	0.1017	0.1804
67.95	0.081	0.9	0.643	0.0518	0.1935	0.3326

Reinforced concrete beam with web reinforcement ratio 0.065%

Element Number North (A8)
Crack Angle 16 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.027	0.002	0.009	0.0070	0.0002	0.0020
45	0.112	0.184	0.232	0.0182	0.0558	0.0522
48	0.002	1.235	0.877	-0.0103	0.3196	-0.1075
51	-0.004	0.647	0.4	0.0010	0.1598	-0.1059
52.846	-0.001	0.32	0.226	-0.0030	0.0827	-0.0287
53.94	0.033	0.432	0.333	0.0025	0.1137	-0.0205
55.2	-0.009	0.393	0.316	-0.0110	0.1070	-0.0014
56.64	0.018	0.494	0.384	-0.0034	0.1314	-0.0176

Element Number North (A8)
Crack Angle 25 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.027	0.002	0.009	0.0067	0.0006	0.0060
45	0.112	0.184	0.232	0.0151	0.0589	0.0264
48	0.002	1.235	0.877	0.0060	0.3032	-0.3061
51	-0.004	0.647	0.4	0.0130	0.1477	-0.1989
52.846	-0.001	0.32	0.226	0.0013	0.0784	-0.0802
53.94	0.033	0.432	0.333	0.0068	0.1094	-0.0882
55.2	-0.009	0.393	0.316	-0.0080	0.1040	-0.0743
56.64	0.018	0.494	0.384	0.0012	0.1268	-0.1000

Element Number North (AB67)
Crack Angle 35 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
45	0	0	0	0.0000	0.0000	0.0000
48	-0.009	1.63	1.374	0.0002	0.4051	-0.5773
51	-0.011	0.77	0.3	-0.0055	0.3850	-0.3941
52.846	0.021	0.414	0.323	0.0105	0.2070	-0.1486
53.94	0.008	0.522	0.376	0.0040	0.2610	-0.2035
55.2	0.047	0.557	0.419	0.0235	0.2785	-0.1996

Reinforced concrete beam with web reinforcement ratio 0.065%

Element Number North (B6)
Crack Angle 36 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	*x*	*y*	*xy*
42	0.022	-0.046	0.024	-0.0089	0.0029	0.0435
45	0.131	0.36	0.361	0.0251	0.0977	-0.0732
48	0.183	1.538	1.253	0.0695	0.3608	-0.5231
51	0.075	0.807	0.585	0.0477	0.1728	-0.3036
52.846	0.039	0.39	0.307	0.0181	0.0892	-0.1383
53.94	0.04	0.524	0.394	0.0252	0.1158	-0.1955
55.2	0.021	0.49	0.369	0.2450	0.1845	-0.1879
56.64	0.06	0.461	0.4	0.0165	0.1138	-0.1476

Element Number North (C5)
Crack Angle 31 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	*x*	*y*	*xy*
42	0.008	-0.004	0.027	-0.0043	0.0053	0.0170
45	0.117	0.203	0.335	-0.0037	0.0837	0.0442
48	1	1.566	1.187	0.3087	0.3328	-0.2949
51	0.057	0.744	0.545	0.0279	0.1723	-0.2355
52.846	0.022	0.386	0.295	0.0096	0.0924	-0.1180
53.94	0.033	0.511	0.376	0.0170	0.1190	-0.1622
55.2	0.023	0.494	0.36	0.0146	0.1147	-0.1603

Element Number North (C5)
Crack Angle 37 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	*x*	*y*	*xy*
42	0.008	-0.004	0.027	-0.0051	0.0061	0.0127
45	0.117	0.203	0.335	-0.0050	0.0850	0.0069
48	1	1.566	1.187	0.3243	0.3172	-0.2985
51	0.057	0.744	0.545	0.0417	0.1585	-0.2904
52.846	0.022	0.386	0.295	0.0166	0.0854	-0.1499
53.94	0.033	0.511	0.376	0.0265	0.1095	-0.2011
55.2	0.023	0.494	0.36	0.0240	0.1052	-0.1984

Reinforced concrete beam with web reinforcement ratio 0.065%

Element Number South (A9)
Crack Angle 20 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
45	0	0	0	0.0000	0.0000	0.0000
48	-0.094	0.724	0.606	-0.0463	0.2038	0.0400
51	-0.004	0.205	0.342	-0.0337	0.0839	-0.1178
52.846	-0.006	0.177	0.141	-0.0051	0.0478	0.0163
53.94	-0.014	0.269	0.25	-0.0149	0.0787	-0.0029
55.2	-0.024	0.259	0.207	-0.0121	0.0709	0.0224

Element Number South (A8)
Crack Angle 13 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
45	0	0	0	0.0000	0.0000	0.0000
48	0.244	1.1903	1.066	0.0347	0.3238	-0.1061
51	0.094	0.6737	0.514	0.0166	0.1754	0.0101
52.846	0.054	0.316	0.271	0.0074	0.0851	-0.0199
53.94	0.06	0.431	0.352	0.0080	0.1147	-0.0144
55.2	0.063	0.413	0.324	0.0108	0.1082	-0.0006

Element Number South (AB78)
Crack Angle 10 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.283	-0.459	-3.467	0.3541	-0.3981	3.0483
45	-0.078	0.452	3.925	-0.3351	0.4286	-3.4219
48	0.194	1.661	1.291	0.0285	0.4353	-0.0907
51	0.1	0.758	0.598	0.0155	0.1990	-0.0463
52.846	0.028	0.368	0.013	0.0254	0.0736	0.2320
53.94	0.06	0.523	0.4	0.0092	0.1365	-0.0228
55.2	0.045	0.486	0.363	0.0062	0.1265	-0.0162

Reinforced concrete beam with web reinforcement ratio 0.065%

Element Number South (B7)
Crack Angle 43 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.009	0.043	-2.6	0.6611	-0.6481	0.2001
45	0.174	0.4	3.062	-0.6223	0.7658	-0.0808
48	0.153	1.628	1.28	0.1126	0.3326	0.7085
52.846	0.08	0.395	0.315	0.0373	0.0814	0.1517
53.94	0.088	0.524	0.4	0.0493	0.1037	0.2109
55.2	0.087	0.484	0.359	0.0496	0.0932	0.1929

Element Number South (B6)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.033	0.04	0.067	0.0021	0.0162	-0.0122
45	-0.019	0.427	0.422	-0.0241	0.1261	0.0841
48	0.015	1.589	1.25	0.0051	0.3959	0.4576
51	0.011	0.755	0.579	0.0068	0.1847	0.2242
52.846	-0.015	0.407	0.313	-0.0027	0.1007	0.1242
53.94	0.004	0.51	0.422	-0.0031	0.1316	0.1366
55.2	-0.005	0.49	0.115	0.0573	0.0640	0.2781

Element Number South (C5)
Crack Angle 13 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.029	0.033	0.042	0.0061	0.0094	-0.0090
45	0.111	0.376	0.354	0.0190	0.1028	-0.0412
48	0.112	1.545	1.168	0.0089	0.4053	0.0090
51	0.057	0.736	0.554	0.0056	0.1927	0.0073
52.846	0.029	0.397	0.299	0.0025	0.1040	0.0034
53.94	0.038	0.508	1.384	-0.1063	0.2428	-0.8955
55.2	0.029	0.487	-0.64	0.1115	0.0175	0.9075

Reinforced concrete beam with web reinforcement ratio 0.065%

Element Number South (C5)
Crack Angle 55 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	x	y	xy
42	0.029	0.033	0.042	0.0053	0.0102	0.0056
45	0.111	0.376	0.354	0.0462	0.0755	0.1623
48	0.112	1.545	1.168	0.1886	0.2256	0.7894
51	0.057	0.736	0.554	0.0912	0.1071	0.3729
52.846	0.029	0.397	0.299	0.0488	0.0577	0.2023
53.94	0.038	0.508	1.384	-0.1727	0.3092	0.6008
55.2	0.029	0.487	-0.64	0.2950	-0.1660	-0.0919

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number North (A9)
Crack Angle 5 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.022	0.179	0.048	-0.0070	-0.0193	0.0597
44.663	-0.109	0.129	0.245	-0.0370	0.0420	0.2108
45.664	-0.235	0.511	-0.764	-0.0182	0.0872	-0.9531
46.498	0.345	-0.163	1.08	0.0424	0.0031	1.0181
47.498	0.163	0.088	0.084	0.0424	0.0203	-0.0344
48.366	0.144	0.062	0.059	0.0378	0.0137	-0.0362
49.3	-0.068	0.142	0.137	-0.0209	0.0394	0.0802
50.2	0.07	0.186	0.124	0.0179	0.0461	-0.0140
51.101	0.037	0.063	0.087	0.0077	0.0173	0.0342
52.202	0.028	0.088	0.083	0.0060	0.0230	0.0194
53.469	0.135	0.072	0.047	0.0361	0.0157	-0.0502
55.204	-0.057	0.168	0.174	-0.0190	0.0467	0.0972
56.538	0.148	0.263	0.224	0.0364	0.0663	0.0082

Element Number North (A8)
Crack Angle 16 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.006	0.219	0.096	0.0077	0.0485	-0.0704
44.663	0.857	0.411	0.523	0.2205	0.0965	0.0240
45.664	-0.742	-0.002	1.542	-0.4250	0.2390	1.4271
46.498	0.026	0.653	-1.082	0.2067	-0.0370	-1.3716
47.498	0.074	0.177	0.161	0.0158	0.0470	0.0028
48.366	-0.077	0.136	0.109	-0.0257	0.0405	0.0110
49.3	-0.086	0.196	0.264	-0.0438	0.0713	0.1025
50.2	0.084	0.216	0.098	0.0304	0.0446	-0.0791
51.101	-0.05	0.085	0.105	-0.0215	0.0303	0.0384
52.202	0.015	0.206	0.155	0.0015	0.0538	-0.0129
53.469	0.014	0.055	0.085	-0.0024	0.0197	0.0320
55.204	-0.001	0.325	0.235	-0.0037	0.0847	-0.0245
56.538	-0.054	-0.702	0.274	-0.1122	-0.0768	0.7246

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number North (B7)
Crack Angle 33 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.096	0.058	0.222	-0.0119	0.0504	0.0763
44.663	0.147	0.56	0.499	0.0341	0.1426	-0.1295
45.664	0.035	0.352	0.333	0.0004	0.0964	-0.0881
46.498	0.089	0.264	0.212	0.0271	0.0611	-0.0655
47.498	0.019	0.139	0.093	0.0105	0.0290	-0.0491
48.366	-0.018	0.115	0.005	0.0153	0.0090	-0.0784
49.3	-0.03	0.22	0.255	-0.0255	0.0730	-0.0491
50.2	0.058	0.193	0.157	0.0173	0.0454	-0.0489
51.101	0.097	0.144	0.126	0.0265	0.0338	-0.0192
52.202	-0.045	0.148	0.053	0.0027	0.0230	-0.0875
53.469	0.075	0.116	0.195	-0.0009	0.0487	0.0217
55.204	-0.016	0.276	-0.085	0.0668	-0.0018	-0.2208
56.538	0.63	0.33	0.572	0.1142	0.1258	0.1745

Element Number North (B7)
Crack Angle 42 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.096	0.058	0.222	-0.0163	0.0548	0.0341
44.663	0.147	0.56	0.499	0.0468	0.1299	-0.1902
45.664	0.035	0.352	0.333	0.0095	0.0872	-0.1430
46.498	0.089	0.264	0.212	0.0330	0.0552	-0.0833
47.498	0.019	0.139	0.093	0.0147	0.0248	-0.0582
48.366	-0.018	0.115	0.005	0.0212	0.0030	-0.0707
49.3	-0.03	0.22	0.255	-0.0193	0.0668	-0.1076
50.2	0.058	0.193	0.157	0.0218	0.0410	-0.0638
51.101	0.097	0.144	0.126	0.0281	0.0321	-0.0228
52.202	-0.045	0.148	0.053	0.0100	0.0158	-0.0958
53.469	0.075	0.116	0.195	-0.0014	0.0491	-0.0100
55.204	-0.016	0.276	-0.085	0.0821	-0.0171	-0.1677
56.538	0.63	0.33	0.572	0.1010	0.1390	0.1588

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number North (C6)
Crack Angle 20 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	x	y	xy
42	0	0	0	0.0000	0.0000	0.0000
44.663	0.006	0.581	0.39	0.0028	0.1439	-0.1109
45.664	0.063	0.313	0.307	0.0039	0.0901	0.0108
46.498	-0.005	0.176	0.279	-0.0271	0.0698	0.0901
47.498	-0.013	0.234	0.02	0.0185	0.0367	-0.1487
48.366	-0.173	0.15	0.166	-0.0623	0.0566	0.0322
49.3	0.151	0.201	0.127	0.0471	0.0409	-0.0536
50.2	-0.076	0.359	0.024	0.0126	0.0581	-0.2298
51.101	0.005	-0.023	0.01	-0.0026	-0.0019	0.0236
53.469	-0.062	0.109	-0.082	0.0065	0.0053	-0.1358
55.204	0.13	0.291	0.266	0.0283	0.0770	-0.0092
56.538	0.005	0.252	0.436	-0.0409	0.1052	0.1562

Element Number North (C5)
Crack Angle 47 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a_1	a_2	a_3	x	y	xy
42	0.115	0.068	0.154	0.0069	0.0389	0.0191
44.663	0.065	0.506	0.419	0.0419	0.1008	-0.2293
45.664	0.225	0.372	0.253	0.0873	0.0620	-0.0701
46.498	-0.061	0.191	0.178	-0.0097	0.0422	-0.1336
47.498	0.058	0.119	0.091	0.0220	0.0222	-0.0306
48.366	-0.168	0.092	0.171	-0.0594	0.0404	-0.1443
49.3	0.17	0.127	0.063	0.0581	0.0162	0.0274
50.2	-0.122	0.254	0.118	0.0068	0.0262	-0.1912
51.101	0.19	0.167	0.285	0.0179	0.0714	0.0040
52.202	-0.105	0.116	-0.052	0.0176	-0.0149	-0.1062
53.469	-0.054	0.124	0.094	-0.0044	0.0219	-0.0929
55.204	0.037	0.293	0.33	0.0023	0.0802	-0.1392
56.538	0.026	0.264	0.127	0.0428	0.0297	-0.1175

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number South (A9)
Crack Angle 17 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	-0.007	0.009	-0.356	0.0485	-0.0480	0.3004
44.663	-0.001	0.115	0.174	-0.0141	0.0426	-0.0646
45.664	0.038	0.14	0.175	-0.0003	0.0448	-0.0428
46.498	-0.019	0.075	0.447	-0.0613	0.0753	-0.3211
47.498	0.001	0.068	-0.304	0.0490	-0.0318	0.2994
48.366	0.004	0.065	0.05	0.0001	0.0171	0.0042
49.3	0.01	0.14	0.137	-0.0034	0.0409	-0.0151
50.2	0.063	0.106	0.126	0.0109	0.0314	-0.0224
51.101	-0.345	0.097	0.073	-0.1043	0.0423	-0.0397
52.202	0.291	0.085	0.072	0.0846	0.0094	0.0386
53.469	0.006	0.065	0.076	-0.0029	0.0207	-0.0171
55.204	0.013	0.145	0.124	-0.0002	0.0397	-0.0004
56.538	-0.2	0.171	0.177	-0.0688	0.0616	-0.0550

Element Number South (A8)
Crack Angle 17 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.082	0.089	0.092	0.0197	0.0230	-0.0034
44.663	0.131	0.418	0.424	0.0180	0.1193	-0.0437
45.664	0.051	0.566	0.241	0.0332	0.1211	0.2000
46.498	0.074	-0.08	0.185	-0.0111	0.0096	-0.1989
47.498	-0.023	0.119	0.105	-0.0107	0.0347	-0.0076
48.366	0.01	0.102	0.114	-0.0036	0.0316	-0.0224
49.3	0.034	0.23	0.196	0.0037	0.0623	0.0017
50.2	0.03	0.157	0.136	0.0043	0.0425	0.0003
51.101	0.022	0.143	0.136	0.0006	0.0406	-0.0105
52.202	0.016	0.126	0.104	0.0017	0.0338	0.0034
53.469	0.021	0.137	0.109	0.0035	0.0360	0.0076
55.204	0.034	0.24	0.199	0.0042	0.0643	0.0062
56.538	0.041	0.291	0.246	0.0044	0.0786	0.0036

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number South (B7)
Crack Angle 45 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0	0	0	0.0000	0.0000	0.0000
44.663	0.15	0.591	0.535	0.0515	0.1337	0.2205
45.664	0.077	0.312	0.261	0.0320	0.0652	0.1175
46.498	0.36	0.239	0.189	0.1025	0.0473	-0.0605
47.498	-0.307	0.149	0.121	-0.0697	0.0302	0.2280
48.366	0.021	0.14	0.108	0.0133	0.0270	0.0595
49.3	0.03	0.216	0.202	0.0110	0.0505	0.0930
50.2	0.052	0.191	0.12	0.0308	0.0300	0.0695
51.101	0.02	0.122	0.121	0.0053	0.0302	0.0510
52.202	0.015	0.17	0.116	0.0173	0.0290	0.0775
53.469	0.07	0.108	0.118	0.0150	0.0295	0.0190
55.204	-0.022	0.266	0.204	0.0100	0.0510	0.1440
56.538	0.048	0.327	0.266	0.0273	0.0665	0.1395

Element Number South (C6)
Crack Angle 25 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0	0	0	0.0000	0.0000	0.0000
44.663	-0.011	0.637	0.512	-0.0119	0.1684	0.1203
45.664	0.012	0.345	0.266	0.0011	0.0881	0.0713
46.498	-0.014	0.221	0.165	-0.0048	0.0565	0.0505
47.498	0.018	0.136	0.113	0.0029	0.0356	0.0221
48.366	-0.025	0.116	0.101	-0.0106	0.0333	0.0183
49.3	0	0.194	0.171	-0.0055	0.0540	0.0267
50.2	-0.002	0.074	0.117	-0.0126	0.0306	-0.0230
51.101	0	0.198	0.106	0.0075	0.0420	0.0713
52.202	-0.001	0.123	0.106	-0.0033	0.0338	0.0186
53.469	0	0.146	0.111	-0.0008	0.0373	0.0315
55.204	-0.003	0.243	0.195	-0.0041	0.0641	0.0460
56.538	0	0.332	0.26	-0.0032	0.0862	0.0667

Reinforced concrete beam with web reinforcement ratio 0.05%

Element Number South (C5)
Crack Angle 40 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	0.044	0.079	0.088	0.0081	0.0227	0.0126
44.663	0.137	0.519	0.468	0.0392	0.1248	0.1638
45.664	0.047	0.297	0.247	0.0191	0.0669	0.1101
46.498	0.04	0.194	0.152	0.0173	0.0412	0.0698
47.498	0.042	0.137	0.103	0.0170	0.0278	0.0444
48.366	0.013	0.164	0.099	0.0163	0.0280	0.0725
49.3	0.031	0.144	0.156	0.0026	0.0412	0.0437
50.2	0.02	0.148	0.111	0.0116	0.0304	0.0583
51.101	0.016	0.133	0.103	0.0091	0.0282	0.0527
52.202	0.017	0.124	0.102	0.0075	0.0277	0.0472
53.469	0.017	0.129	0.106	0.0077	0.0288	0.0494
55.204	0.028	0.241	0.184	0.0168	0.0504	0.0963
56.538	0.031	0.319	0.252	0.0185	0.0690	0.1284

Element Number South (C5)
Crack Angle 13 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	0.044	0.079	0.088	0.0085	0.0222	-0.0161
44.663	0.137	0.519	0.468	0.0237	0.1403	-0.0421
45.664	0.047	0.297	0.247	0.0067	0.0793	-0.0126
46.498	0.04	0.194	0.152	0.0081	0.0504	0.0023
47.498	0.042	0.137	0.103	0.0102	0.0345	0.0087
48.366	0.013	0.164	0.099	0.0040	0.0402	0.0237
49.3	0.031	0.144	0.156	0.0017	0.0421	-0.0368
50.2	0.02	0.148	0.111	0.0037	0.0383	0.0038
51.101	0.016	0.133	0.103	0.0024	0.0349	0.0000
52.202	0.017	0.124	0.102	0.0022	0.0331	-0.0049
53.469	0.017	0.129	0.106	0.0021	0.0344	-0.0051
55.204	0.028	0.241	0.184	0.0043	0.0630	0.0022
56.538	0.031	0.319	0.252	0.0030	0.0845	-0.0061

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number North (A8)
Crack Angle 30 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	-0.893	0	0.016	-0.2676	0.0443	-0.1554
45	-0.012	-0.011	-0.031	0.0013	-0.0070	-0.0102
47	0.669	3.049	2.578	0.1603	0.7692	-0.6711
47.9	0.049	0.346	0.243	0.0210	0.0778	-0.1059
48.86	0.041	0.198	0.162	0.0109	0.0489	-0.0467
49.76	0.017	0.204	0.177	0.0015	0.0537	-0.0477
50.7	0.042	0.21	0.14	0.0180	0.0450	-0.0657
51.6	0.011	0.101	0.206	-0.0241	0.0521	0.0360
52.4	0.019	0.234	0.18	0.0066	0.0566	-0.0663
53.3	0.079	0.255	0.182	0.0275	0.0560	-0.0687
54.27	0.002	0.327	0.224	0.0079	0.0743	-0.1110
55.2	0.028	0.28	0.257	0.0004	0.0766	-0.0576

Element Number North (B7)
Crack Angle 23 degree

Applied Load (tf)	**Increment deformation in beam coordinate**			**Increment deformation in cracking coordinate**		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	0.025	0.014	0.021	0.0056	0.0042	0.0050
45	-0.003	0.019	0.019	-0.0019	0.0059	-0.0003
47	0.456	3.268	2.241	0.1532	0.7778	-0.7481
47.9	0.03	0.318	0.15	0.0228	0.0642	-0.1203
48.86	0.033	0.237	0.153	0.0128	0.0547	-0.0609
49.76	0.021	0.209	0.168	0.0029	0.0546	-0.0308
50.7	0.026	0.215	0.167	0.0054	0.0549	-0.0357
51.6	0.017	0.237	0.178	0.0035	0.0600	-0.0437
52.4	0.039	0.278	0.209	0.0098	0.0695	-0.0509
53.3	-0.006	0.257	0.201	-0.0050	0.0678	-0.0421
54.27	0.147	0.284	0.255	0.0349	0.0729	-0.0218
55.2	-0.078	0.261	0.217	-0.0291	0.0749	-0.0347

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number North (B7)
Crack Angle 57 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	0.025	0.014	0.021	0.0040	0.0058	0.0044
45	-0.003	0.019	0.019	0.0006	0.0034	-0.0145
47	0.456	3.268	2.241	0.5219	0.4091	-1.4386
47.9	0.03	0.318	0.15	0.0636	0.0234	-0.1218
48.86	0.033	0.237	0.153	0.0400	0.0275	-0.1005
49.76	0.021	0.209	0.168	0.0262	0.0313	-0.1074
50.7	0.026	0.215	0.167	0.0291	0.0311	-0.1052
51.6	0.017	0.237	0.178	0.0313	0.0322	-0.1212
52.4	0.039	0.278	0.209	0.0402	0.0390	-0.1297
53.3	-0.006	0.257	0.201	0.0275	0.0352	-0.1508
54.27	0.147	0.284	0.255	0.0518	0.0559	-0.0786
55.2	-0.078	0.261	0.217	0.0114	0.0343	-0.2059

Element Number North (C6)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a 2*	*a 3*	*x*	*y*	*xy*
42	-0.004	0.013	0.007	-0.0005	0.0027	-0.0061
45	0.023	0.005	0.02	0.0033	0.0037	0.0108
47	0.357	3.206	2.504	0.1109	0.7799	-0.8724
47.9	0.032	0.309	0.243	0.0096	0.0756	-0.0837
48.86	0.026	0.249	0.164	0.0147	0.0540	-0.0833
49.76	0.026	0.199	0.159	0.0072	0.0490	-0.0517
50.7	0.02	0.232	0.165	0.0098	0.0532	-0.0723
51.6	0.005	0.249	0.174	0.0063	0.0572	-0.0822
52.4	0.008	0.224	0.196	-0.0018	0.0598	-0.0535
53.3	0.049	0.267	0.193	0.0183	0.0607	-0.0769
54.27	-0.009	0.295	0.213	0.0016	0.0699	-0.0966
55.2	0.024	0.283	0.252	0.0009	0.0759	-0.0629

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number North (C6)
Crack Angle 45 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	-0.004	0.013	0.007	-0.0003	0.0030	0.0015
45	0.023	0.005	0.02	0.0020	0.0050	0.0090
47	0.357	3.206	2.504	0.2647	0.6260	-1.4245
47.9	0.032	0.309	0.243	0.0245	0.0608	-0.1385
48.86	0.026	0.249	0.164	0.0277	0.0410	-0.1115
49.76	0.026	0.199	0.159	0.0165	0.0398	-0.0865
50.7	0.02	0.232	0.165	0.0217	0.0413	-0.1060
51.6	0.005	0.249	0.174	0.0200	0.0435	-0.1220
52.4	0.008	0.224	0.196	0.0090	0.0490	-0.1080
53.3	0.049	0.267	0.193	0.0307	0.0483	-0.1090
54.27	-0.009	0.295	0.213	0.0182	0.0533	-0.1520
55.2	0.024	0.283	0.252	0.0137	0.0630	-0.1295

Element Number North (CD56)
Crack Angle 47 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
45	0	0	0	0.0000	0.0000	0.0000
47	-0.059	3.267	2.425	-0.0494	0.8514	-0.4400
47.9	-0.001	0.292	0.248	-0.0082	0.0809	-0.0156
48.86	0.001	0.208	0.144	0.0000	0.0523	-0.0363
49.76	0.002	0.116	0.149	-0.0106	0.0401	0.0323
50.7	-0.004	0.309	0.163	0.0065	0.0698	-0.0926
51.6	-0.003	0.115	0.166	-0.0150	0.0430	0.0463
52.4	0.003	0.244	0.2	-0.0045	0.0662	-0.0189
53.3	-0.027	0.246	0.19	-0.0117	0.0665	-0.0261
54.27	-0.003	0.293	0.221	-0.0043	0.0768	-0.0369
55.2	0.078	0.361	0.155	0.0381	0.0716	-0.1404

Element Number North (CD45)
Crack Angle 20 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
45	0.053	0.0232	0.027	0.0142	0.0049	0.0011
47	0.34	2.7738	2.243	0.0459	0.7325	-0.2566
47.9	0.018	0.298	0.231	0.0010	0.0780	-0.0341
48.86	0.016	0.188	0.156	0.0004	0.0506	-0.0139
49.76	0.017	0.213	0.16	0.0028	0.0547	-0.0285
50.7	0.069	0.202	0.148	0.0191	0.0486	-0.0332
51.6	-0.017	0.186	0.153	-0.0093	0.0516	-0.0128
52.4	0.012	0.266	0.189	0.0024	0.0671	-0.0433
53.3	0.018	0.247	0.18	0.0036	0.0627	-0.0372
54.27	0.033	0.212	0.218	-0.0019	0.0631	0.0156

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number South (A9)
Crack Angle 20 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
46.9	0.511	1.735	1.626	0.0827	0.4788	0.0081
47.9	0.056	0.197	0.165	0.0119	0.0513	0.0158
48.86	0.015	0.325	0.119	0.0210	0.0640	0.1387
49.76	0.025	0.099	0.108	0.0010	0.0300	-0.0115
50.7	0.024	0.071	0.11	-0.0027	0.0264	-0.0328
51.6	0.025	0.062	0.134	-0.0072	0.0290	-0.0574
52.4	0.039	0.348	0.135	0.0282	0.0686	0.1441
53.3	0.023	0.162	0.131	0.0036	0.0426	0.0152
54.27	0.032	0.008	0.172	-0.0171	0.0271	-0.1242
55.2	0.044	0.188	0.208	0.0004	0.0576	-0.0242

Element Number South (A8)
Crack Angle 25 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	0.014	0.005	0.011	0.0028	0.0019	-0.0044
45	0.01	0.029	0.039	-0.0004	0.0101	-0.0053
47	-0.005	2.952	2.528	-0.0712	0.8079	0.4548
47.9	0.006	0.298	0.252	-0.0046	0.0806	0.0476
48.86	-0.03	0.205	0.109	-0.0011	0.0449	0.0762
49.76	0.001	0.2	0.163	-0.0028	0.0531	0.0360
50.7	0.025	0.181	0.161	0.0021	0.0494	0.0225
51.6	-0.026	0.22	0.175	-0.0105	0.0590	0.0441
52.4	0.001	0.218	0.194	-0.0062	0.0610	0.0288
53.3	-0.012	0.244	0.181	-0.0040	0.0620	0.0563
54.27	0.003	0.282	0.236	-0.0047	0.0759	0.0468
55.2	-0.001	0.147	0.219	-0.0216	0.0581	-0.0372

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number <u>South (B7)</u>
Crack Angle <u>45 degree</u>

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
42	-0.014	0	-1.292	0.3195	-0.3230	0.0070
45	0.042	0.019	1.303	-0.3105	0.3258	-0.0115
47	0.521	3.137	2.593	0.2663	0.6482	1.3080
47.9	0.012	0.311	0.246	0.0193	0.0615	0.1495
48.86	0.023	0.212	0.164	0.0178	0.0410	0.0945
49.76	0.053	0.2	0.154	0.0248	0.0385	0.0735
50.7	0.018	0.203	0.173	0.0120	0.0432	0.0925
51.6	0.028	0.233	0.183	0.0195	0.0457	0.1025
52.4	0.03	0.251	0.198	0.0208	0.0495	0.1105
53.3	0.033	0.241	0.178	0.0240	0.0445	0.1040
54.27	0.044	0.302	0.235	0.0278	0.0587	0.1290
55.2	0.027	0.193	0.224	-0.0010	0.0560	0.0830

Element Number <u>South (BC67)</u>
Crack Angle <u>27 degree</u>

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a 1	*a* 2	*a* 3	*x*	*y*	*xy*
45	0	0	0	0.0000	0.0000	0.0000
47	0.621	3.209	2.624	0.1452	0.8123	0.6301
47.9	0.055	0.262	0.203	0.0154	0.0638	0.0576
48.86	0.034	0.204	0.179	0.0051	0.0544	0.0335
49.76	0.044	0.208	0.154	0.0138	0.0492	0.0499
50.7	0.05	0.208	0.169	0.0126	0.0519	0.0404
51.6	0.045	0.235	0.187	0.0115	0.0585	0.0492
52.4	0.052	0.256	0.196	0.0150	0.0620	0.0578
53.3	0.036	0.241	0.191	0.0089	0.0603	0.0521
54.27	0.068	0.269	0.219	0.0171	0.0671	0.0516
55.2	0.059	0.31	5.902	-1.1287	1.2210	-3.2591

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number South (C5)
Crack Angle 18 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	-0.003	0.04	-0.002	0.0033	0.0060	0.0292
45	0.009	-0.02	0.021	-0.0023	-0.0004	-0.0300
47	0.434	3.023	2.443	0.0653	0.7989	0.1828
47.9	0.032	0.31	0.231	0.0058	0.0797	0.0332
48.86	0.018	0.199	0.16	0.0013	0.0530	0.0115
49.76	0.043	0.194	0.15	0.0097	0.0495	0.0189
50.7	0.014	0.204	0.165	-0.0002	0.0547	0.0105
51.6	0.024	0.233	0.175	0.0042	0.0601	0.0238
52.4	0.029	0.243	0.18	0.0059	0.0621	0.0273
53.3	0.026	0.131	0.195	-0.0081	0.0474	-0.0634
54.27	0.035	0.211	0.176	0.0052	0.0563	0.0088
55.2	0.026	0.294	-0.002	0.0367	0.0433	0.2098

Element Number South (C5)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	x	y	xy
42	-0.003	0.04	-0.002	0.0064	0.0029	0.0289
45	0.009	-0.02	0.021	0.0090	-0.0200	0.0210
47	0.434	3.023	2.443	0.1156	0.7486	0.7638
47.9	0.032	0.31	0.231	0.0124	0.0731	0.0904
48.86	0.018	0.199	0.16	0.0047	0.0496	0.0526
49.76	0.043	0.194	0.15	0.0134	0.0459	0.0496
50.7	0.014	0.204	0.165	0.0033	0.0512	0.0543
51.6	0.024	0.233	0.175	0.0090	0.0553	0.0672
52.4	0.029	0.243	0.18	0.0111	0.0569	0.0707
53.3	0.026	0.131	0.195	-0.0122	0.0514	-0.0128
54.27	0.035	0.211	0.176	0.0083	0.0532	0.0497
55.2	0.026	0.294	-0.002	0.0583	0.0217	0.1970

Reinforced concrete beam with web reinforcement ratio 0.035%

Element Number South (C4)
Crack Angle 30 degree

Applied Load (tf)	Increment deformation in beam coordinate			Increment deformation in cracking coordinate		
	a_1	a_2	a_3	*x*	*y*	*xy*
42	0.023	0.022	0.003	0.0099	0.0013	0.0093
45	-0.018	0.016	0.038	-0.0108	0.0103	-0.0048
47	-0.009	2.764	2.262	-0.0204	0.7092	0.7585
47.9	-0.009	0.284	0.209	0.0006	0.0682	0.0911
48.86	-0.018	0.2	0.156	-0.0049	0.0504	0.0619
49.76	0.007	0.189	0.147	0.0025	0.0465	0.0543
50.7	0.019	0.195	0.157	0.0049	0.0486	0.0512
51.6	0.025	0.22	0.167	0.0088	0.0524	0.0622
52.4	-0.043	0.235	0.182	-0.0120	0.0600	0.0774
53.3	-0.005	0.224	0.175	-0.0011	0.0559	0.0664
54.27	0.009	0.287	0.228	0.0023	0.0717	0.0804
55.2	0.001	0.37	0.165	0.0278	0.0650	0.1700