Relativistic Tunneling Anomaly and Singular Force

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Dirac Dynamics Now

- Irrelevant?
  Strong fields in soft matter
- Obsolete?
  Dirac sea Monster can be tamed
  - Klein paradox, Plesset paradoxes
    - Krekola et al. (2004), Gichetti+Sorace (2007)
- Trivial?
  Useful for exotic low energy phenomena

1D Dirac Equation

- $$\left[\frac{\varepsilon}{c} - \vec{\alpha}\cdot\vec{p} - \beta mc\right]\Psi = 0$$
- Two component in 1D (no spin)

$$\begin{pmatrix}\varphi' \\ \chi'\end{pmatrix} = \begin{pmatrix} 0 & m+\varepsilon+S-V \\ m-\varepsilon+S+V & 0\end{pmatrix}\begin{pmatrix}\varphi \\ \chi\end{pmatrix}$$

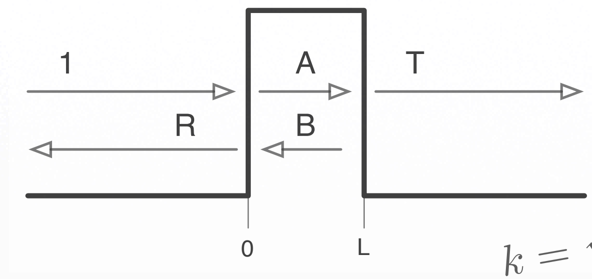
$$w = \varepsilon - m$$

- Potentials with different Lorentz covariance
  - *S* : scaler   *V* : vector (temporal component)

Barrier Scatterings

- Potential barriers

$$V(x) = v\,\Theta(x)\Theta(L-x)$$
$$S(x) = s\,\Theta(x)\Theta(L-x)$$



- Scattering problem

$$k = \sqrt{\varepsilon^2 - m^2}$$
$$p = \sqrt{(\varepsilon - v)^2 - (m+s)^2}$$

$$\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = \begin{pmatrix}1\\ \frac{ik}{m+\varepsilon}\end{pmatrix} e^{ikx} - R\begin{pmatrix}1\\ \frac{-ik}{m+\varepsilon}\end{pmatrix} e^{-ikx} \qquad (x<0)$$

$$\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = A\begin{pmatrix}1\\ \frac{ip}{m+\varepsilon+s-v}\end{pmatrix} e^{ipx} - B\begin{pmatrix}1\\ \frac{-ip}{m+\varepsilon+s-v}\end{pmatrix} e^{-ipx} \qquad (0<x<L)$$

$$\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = T\begin{pmatrix}1\\ \frac{ik}{m+\varepsilon}\end{pmatrix} e^{ikx} \qquad (L<x)$$

Transmission & Reflection

- Amplitudes *T*, *R* given by

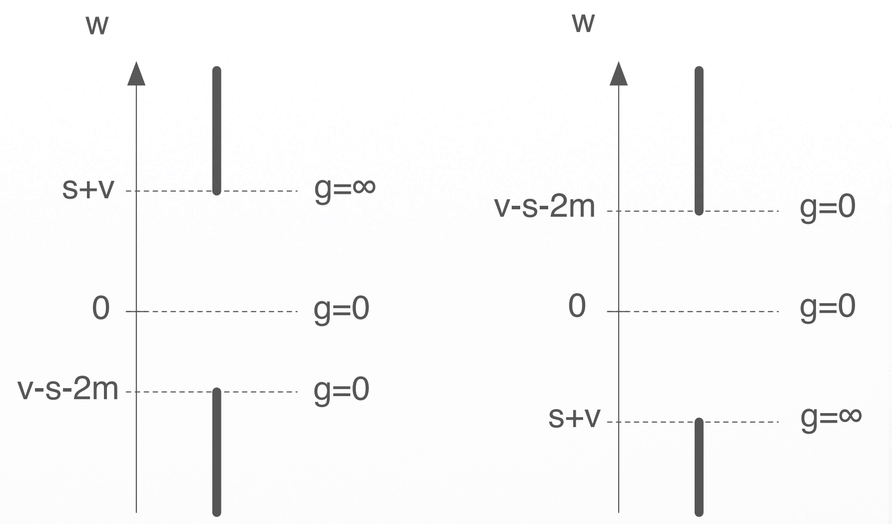
$$T = \frac{e^{-ikL}}{\cos pL - \frac{i}{2}(1/g+g)\sin pL},$$

$$R = \frac{-\frac{i}{2}(1/g-g)\sin pL}{\cos pL - \frac{i}{2}(1/g+g)\sin pL}.$$

- $g = \sqrt{\frac{w(w+2m+s-v)}{(w+2m)(w-s-v)}Q}$

- Giachetti-Sorace factor

$$Q = 1 - \Theta(v-s-2m-w)\Theta(s+v-w)$$



No-penetration for negative continuous states

$$\varphi(0_-) = 0, \quad \chi(0_-) = const$$

Zero-Range Potential

- Assume that potential range *L* is small

$$v = \frac{\bar{v}}{L}, \quad s = \frac{\bar{s}}{L}, \quad (L \to 0), \qquad K = \sqrt{\frac{w}{w+2m}}$$

- $$T = \frac{1}{\alpha + \frac{i}{2}[u_+/K - u_- K]}, \; R = \frac{\frac{i}{2}[u_+/K + u_- K]}{\alpha + \frac{i}{2}[u_+/K - u_- K]},$$

- $$u_+ = (\bar{s} + \bar{v})\frac{\sin\beta}{\beta}, \quad u_- = (\bar{s} - \bar{v})\frac{\sin\beta}{\beta} \qquad \beta = \sqrt{\bar{v}^2 - \bar{s}^2}$$

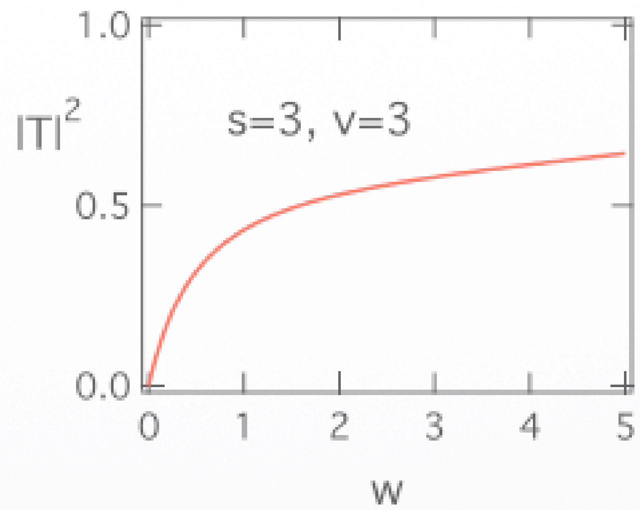
Conventional Limit

- Delta limit : $v = s$

$$T = \frac{1}{1 + i\bar{v}/K}$$

$$R = \frac{i\bar{v}/K}{1 + i\bar{v}/K}$$

$$K \approx k/(2m) \quad \text{if} \quad w \approx k^2/(2m)$$



- Dirac's delta function in non-rel. scatterings
  --> high-pass quantum filter

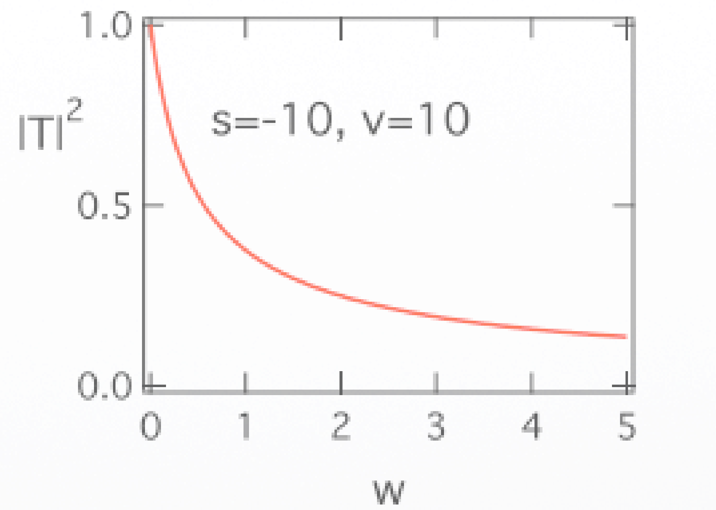
Exotic Limit

- Delta-prime limit : $\bar{s} = -\bar{v}$

$$T = \frac{1}{1 + i\bar{v}K}$$

$$R = \frac{-i\bar{v}K}{1 + i\bar{v}K}$$

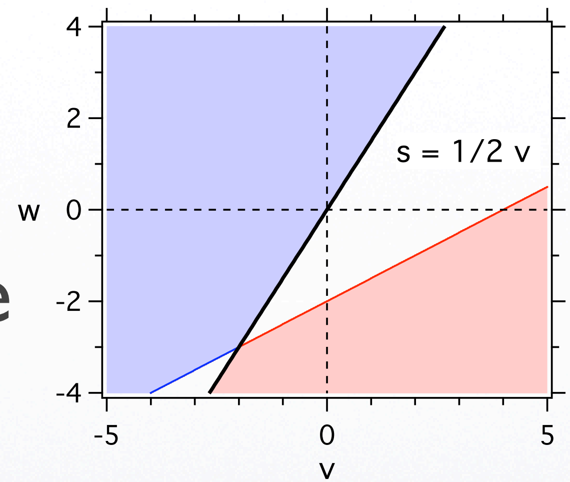
$$K \approx k/(2m) \quad \text{if} \quad w \approx k^2/(2m)$$

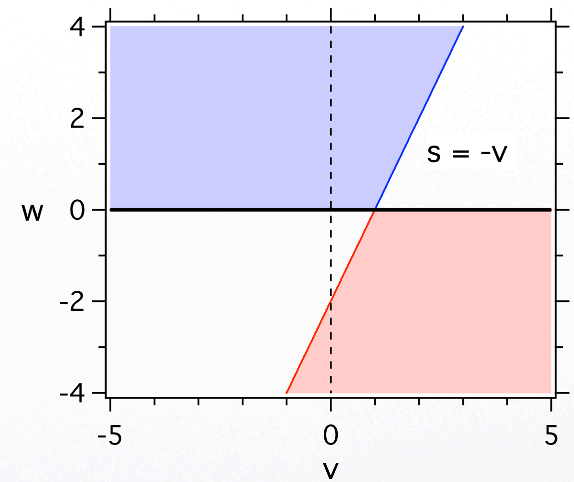


- Delta-prime contact force (hard to come-by) --> low-pass quantum filter

Dirac Spectra

- Function g
  is zero at ---
  is infty at ---
  for generic case





- g is finite at ---
  for $|\bar{s}|^2 = |\bar{v}|^2$

$$g = \sqrt{\frac{w + 2m - 2v}{w + 2m}} = \frac{1}{i}\sqrt{\frac{2v - w - 2m}{w + 2m}}$$

- Long-range tunneling near both particle threshold

Schrödinger Limit

- For w << m, we have Schrodinger-like eq.

$$-\frac{d}{dx}\frac{1}{2m^*}\frac{d}{dx}\varphi + U\varphi = w\varphi,$$

- Efective mass and "non-relativistic" potential

$$m^* = m + \frac{w}{2} + \frac{S-V}{2}, \quad U = S + V.$$

- True non-relativistic limit is obtained when potentials *S* and *V* are also small
- Realizable for particle in heterotic medium(?)

Summary

- Conventional view that Dirac equation *per se* is irrelevant and obsolete probably is wrong

- Dirac dynamics can show up in low energy
- Anomalous threshold tunneling low pass filter for "*S*=-*V*" potential

Ref: P.Hejcik & T.Cheon, Europhys. Lett. 81 (2008) 50001