Trophic Pyramid in Ecosystems

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Introduction

- Universal characteristics of Ecosystems
  : in dominance/competition/hierarchy
- Behavioral choice with Preference
  : dynamics with two Time Scales
- Lotka-Volterra with Parametric Variation
- Quantitative Predictions w/o ad hoc assumption

Pray-Predator System

- Lotka-Volterra eq. for N=2 Food web

  $1/x\ dx/dt = b\ x - a\ x^2 - R\ x\ y$

  $1/y\ dy/dt = -d\ y + f\ R\ x\ y$

- (a, b, d, f) fixed parameters

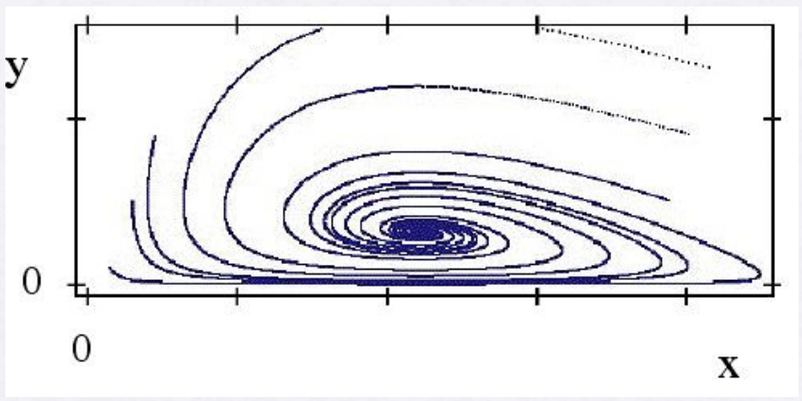
- aggression {R} varies adiabatically to maximize predator y

Fixed Point Solution

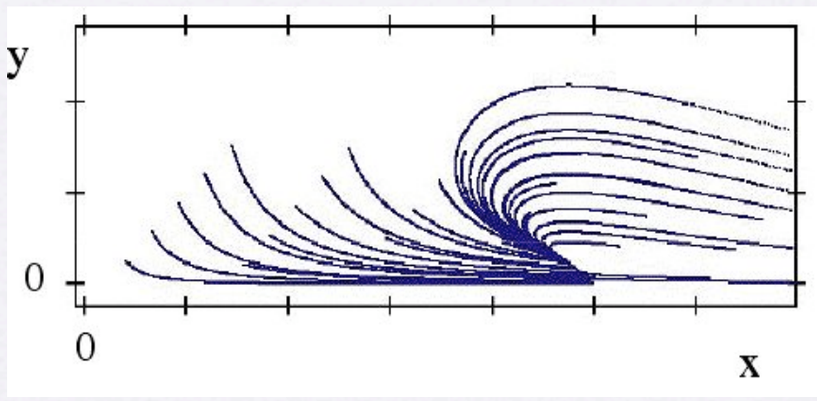
- System evolves toward (X, Y) :

$$0 = b - a X - R Y$$

$$0 = -d + f R X$$

- 

"right" R



"wrong" R

Optimal Aggression

- Fixed point (X[R], Y[R])
  stability eigenvalues <0 (fRb >> ad)

- Maximize Y[R] = -b/R + ad/fR^2

- Evol. stability at $R^* = 2ad/fb$

  $X^* = b/2a, \quad Y^* = b/2R^*$

- half of natural population X=b/a taken

Tritrophic System

- N=3 Food web

  $1/x\ dx/dt = b\ x - a\ x^2 - R_2\ x\ y$

  $1/y\ dy/dt = -d_2\ y + f_2\ R_2\ x\ y - R_3\ y\ z$

  $1/z\ dz/dt = -d_3\ z + f_3\ R_3\ y\ z$

- Fixed point (X,Y,Z)

- Maximize Z[R_3], Y[R_2] simultaneously

Separation to 2 Systems

- $0 = b/2 - a\,(X - b/2a) - R_2\,Y$
  $0 = -d_2/2 + f_2\,R_2\,(X - b/2a)$

- $0 = b_2 - a_2\,Y - R_3\,Z$
  $0 = -d_3 + f_3\,R_3\,Y$

  $a_2 := f_2 R_2\text{^}2/a$
  $b_2 := f_2 b R_2/a - d_2$

- $R_2^* = 2ad_2/f_2b$, $X^* = 3b/4a$, $Y^* = b/2R_2^*$
  $R_3^* = 8ad_2d_3/f_2f_3b\text{^}2$, $Z^* = d_2/2R_3$

Pyramidal Hierarchy

- N-trophic food web chain $\{x_1, x_2, ..., x_N\}$
- Fibonacci : eg. N=5,

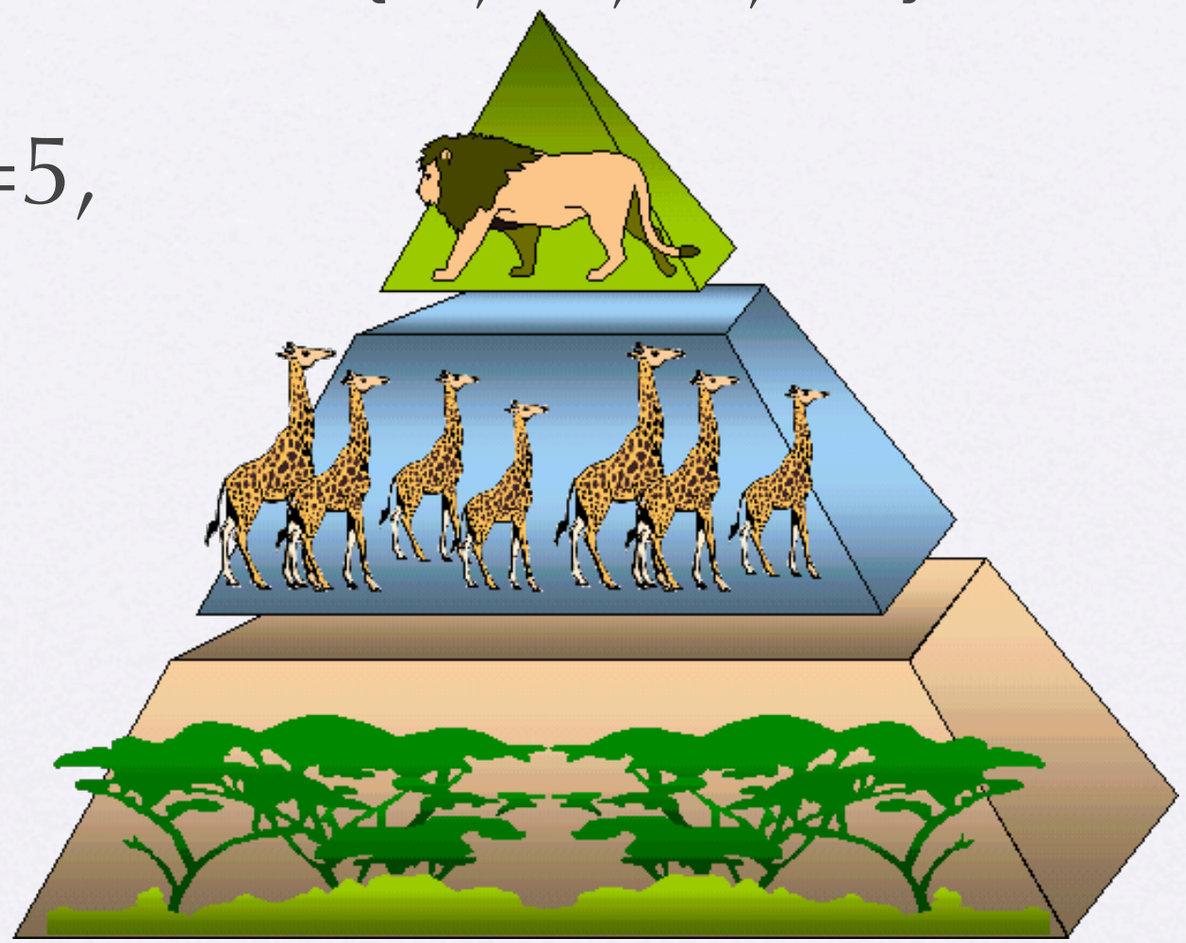
$X_5^* \sim (1/256)\ X_1^*$

$X_4^* \sim (1/128)\ X_1^*$

$X_3^* \sim (3/64)\ X_1^*$

$X_2^* \sim (5/32)\ X_1^*$

$X_1^* = (11/16)\ b/a$



Shape of Ecosystems

- Multidimensional Lotka-Volterra eq. with adiabatic parameter shift
- Elton’s Universal Pyramid
- Extensions to multi-species in a trophic level : Stabilization by Dominance

References

- T. Cheon, “Evolutionary stability of ecological hierarchy”, Phys. Rev. Lett. 90 (2003) 258105(4).

- T. Cheon and S. Ohta, “Suppression of ecological competition by apex predator”, cond-mat/0305351 (2003).

- http://www.mech.kochi-tech.ac.jp/cheon/