Superfluidity of Disordered Bose Systems: Numerical Analysis of the Gross-Pitaevskii Equation with a Random Potential

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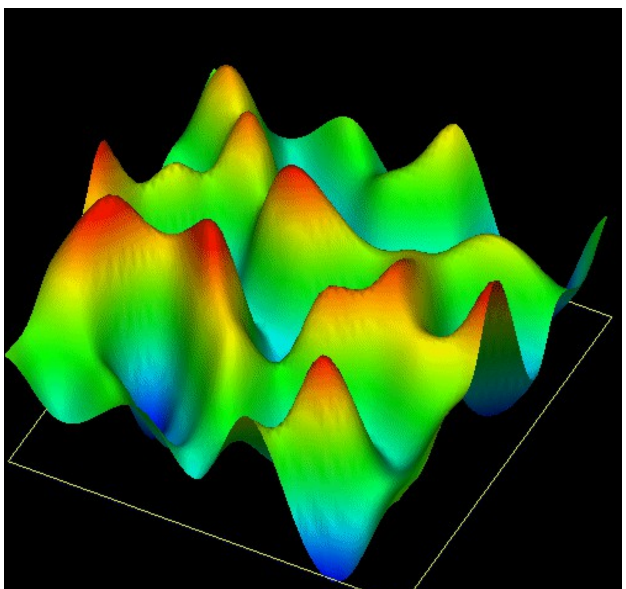
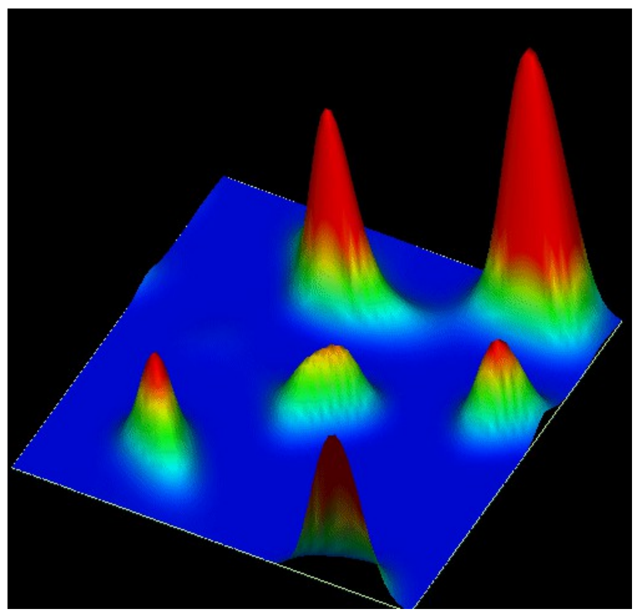
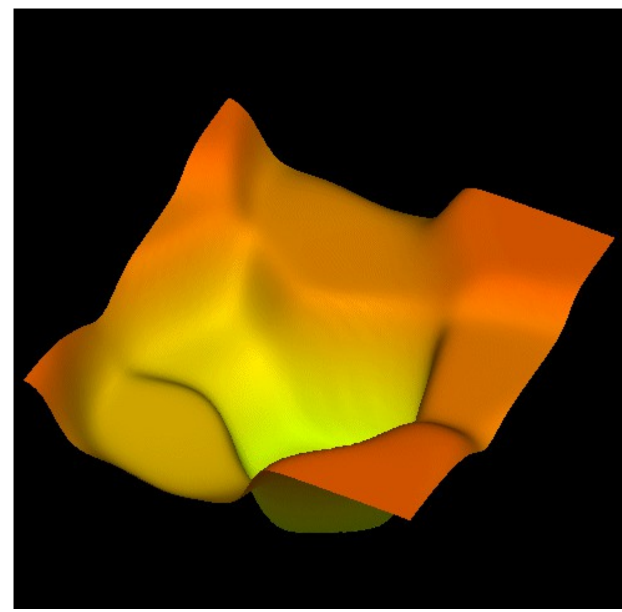






Motivation

- **How does disorder affect Bose-Einstein condensation and its Superfluidity?**
- **What is the relation between BEC and superfluidity?**
- **By adjusting with disorder, we may divide BEC from superfluidity and understand this relation!**
- **We investigate this problem by the Gross-Pitaevskii(GP) equation with a random potential**

Dynamics of two dimensional Bose system

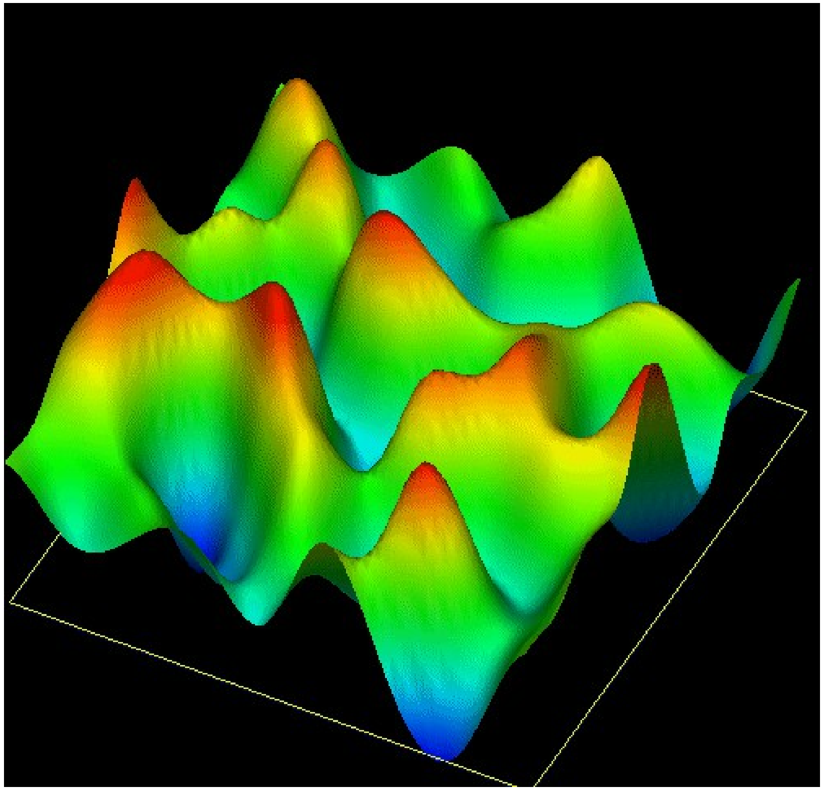
Bose field operator →macroscopic wave function of BEC and its fluctuation $\hat{\Psi}(\vec{x}) = \Phi(\vec{x}) + \hat{\varphi}(\vec{x})$

neglecting the fluctuation ⇒equation of the macroscopic wave function(GP equation)

$$i\hbar \frac{\partial}{\partial t}\Phi(\vec{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + U(\vec{x}) + g\left|\Phi(\vec{x},t)\right|^2 \right]\Phi(\vec{x},t)$$

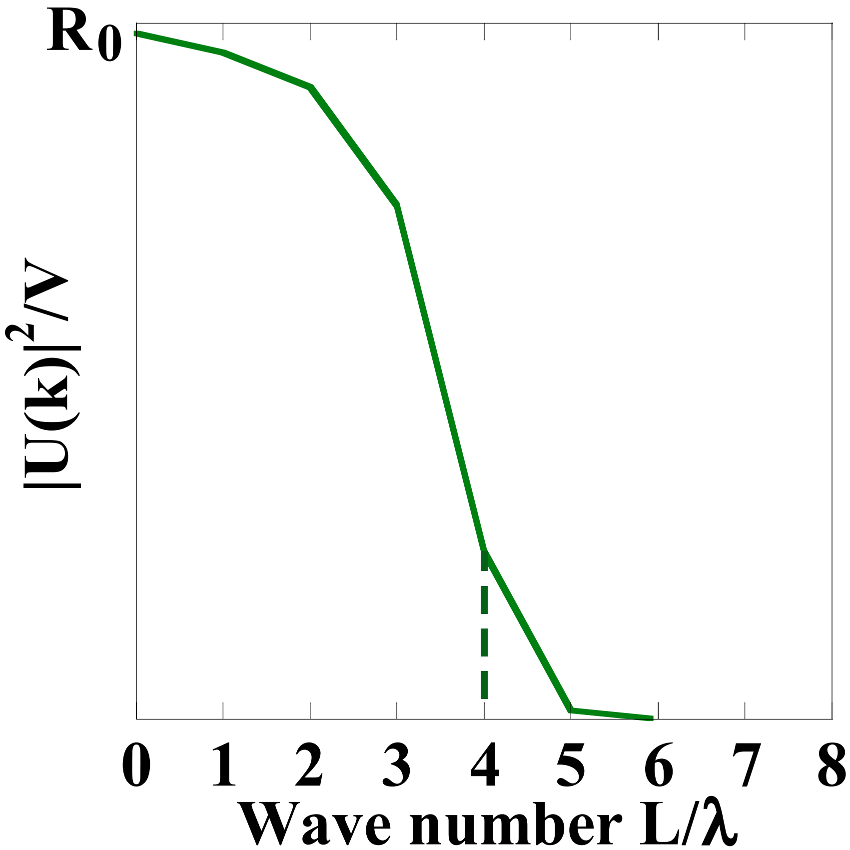
U(x) : random potential

One example of U(x)



The average number of wave is regulated to be about 4 in a direction.

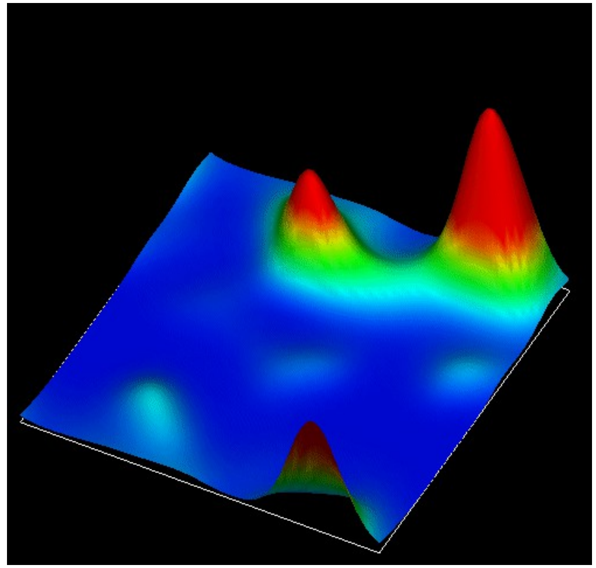
Fourier transformation of U(x)
(we take 100 ensemble average)



Of course, U(k) decays above the wave number 4

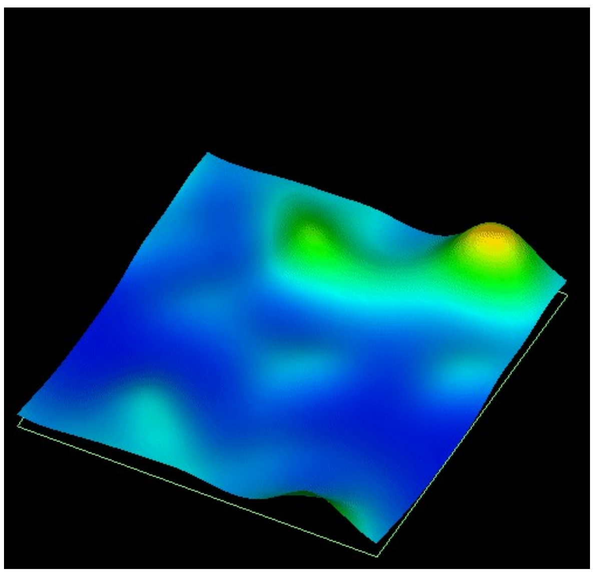
Ground state of GP equation

Parameter: strength of the random potential R_0 .

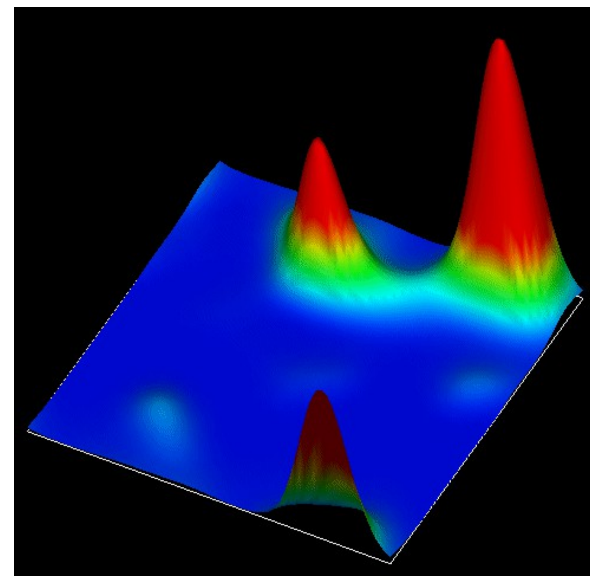


$R_0 / \mu = 50$

$R_0 / \mu = 20$



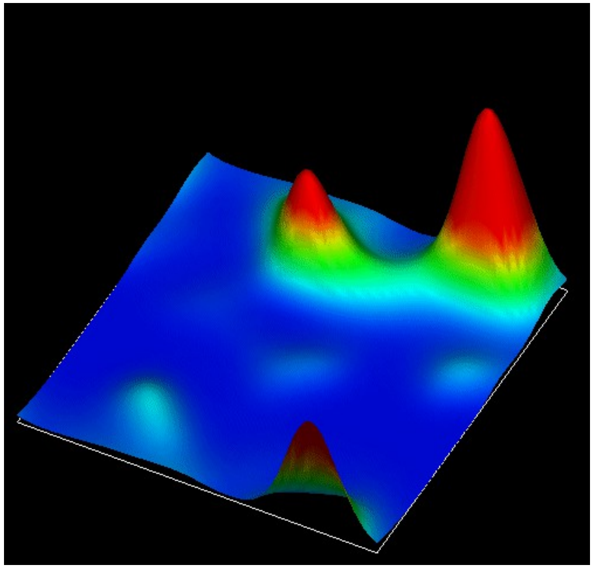
$R_0 / \mu = 70$



Wave function localizes as R_0 increases

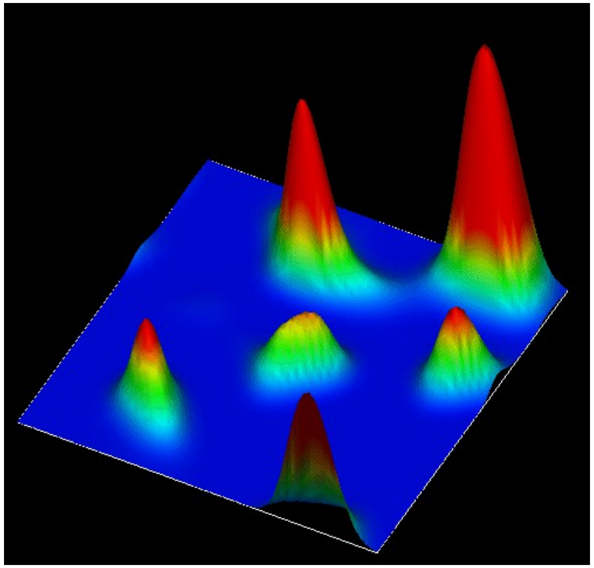
Parameter: coherence length ξ of the wave function. $\xi = \hbar / \sqrt{2m\mu} \quad (\mu \sim gn)$

λ: Characteristic width of the random potential

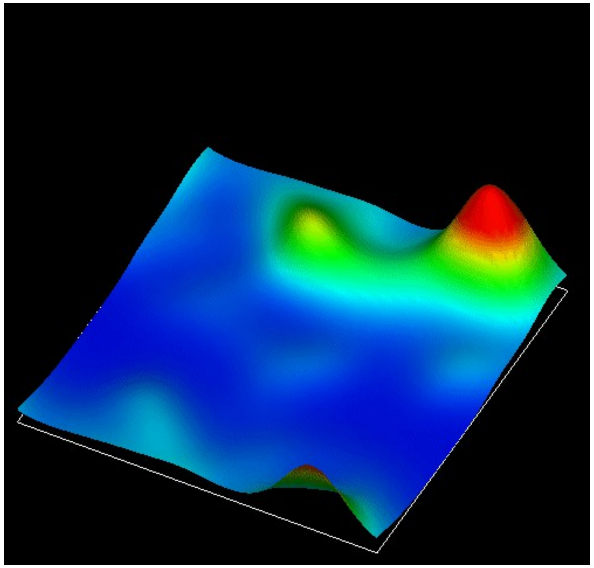


$\xi / \lambda = 2$

$\xi / \lambda = 1$

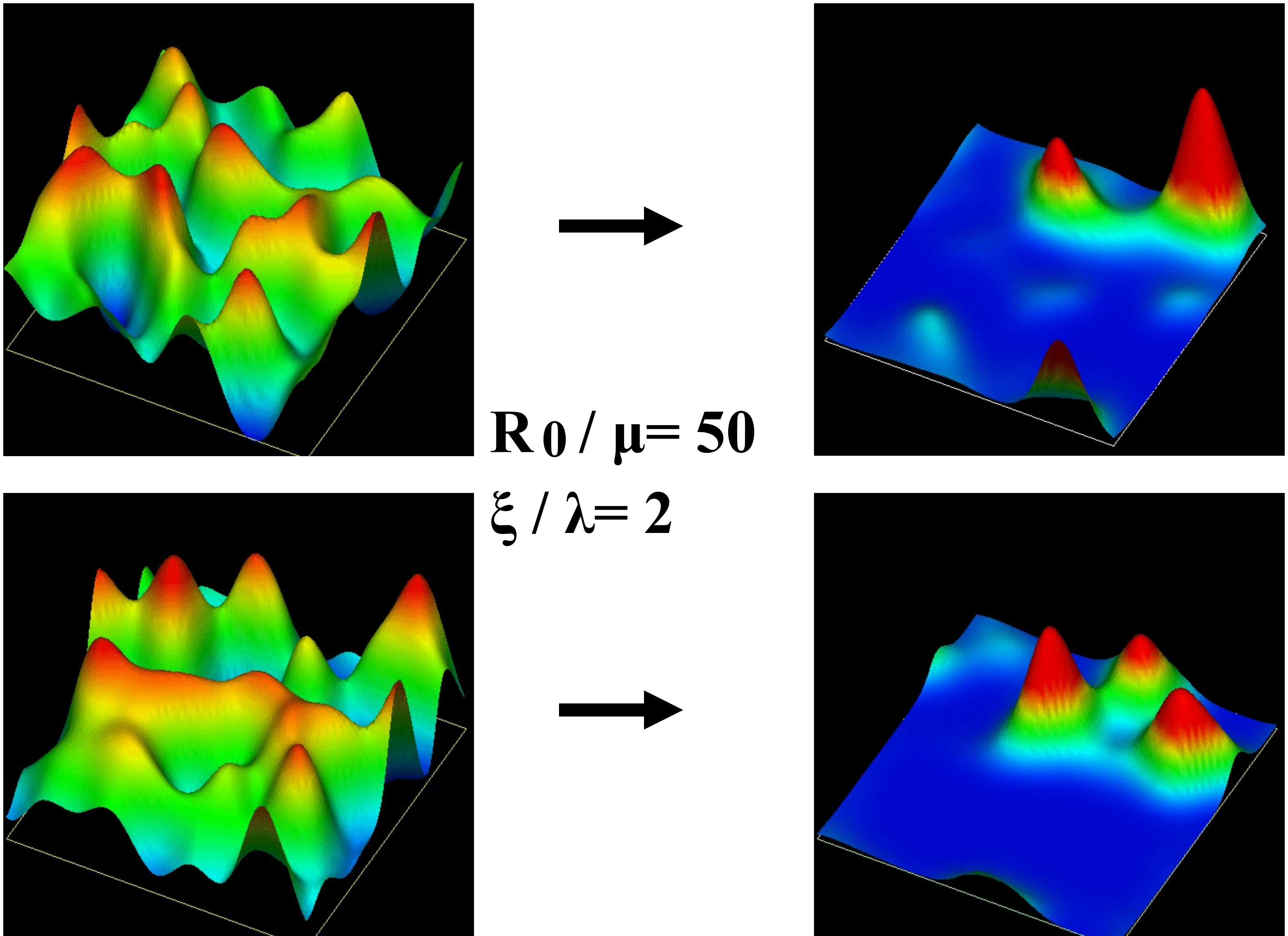


$\xi / \lambda = 3$



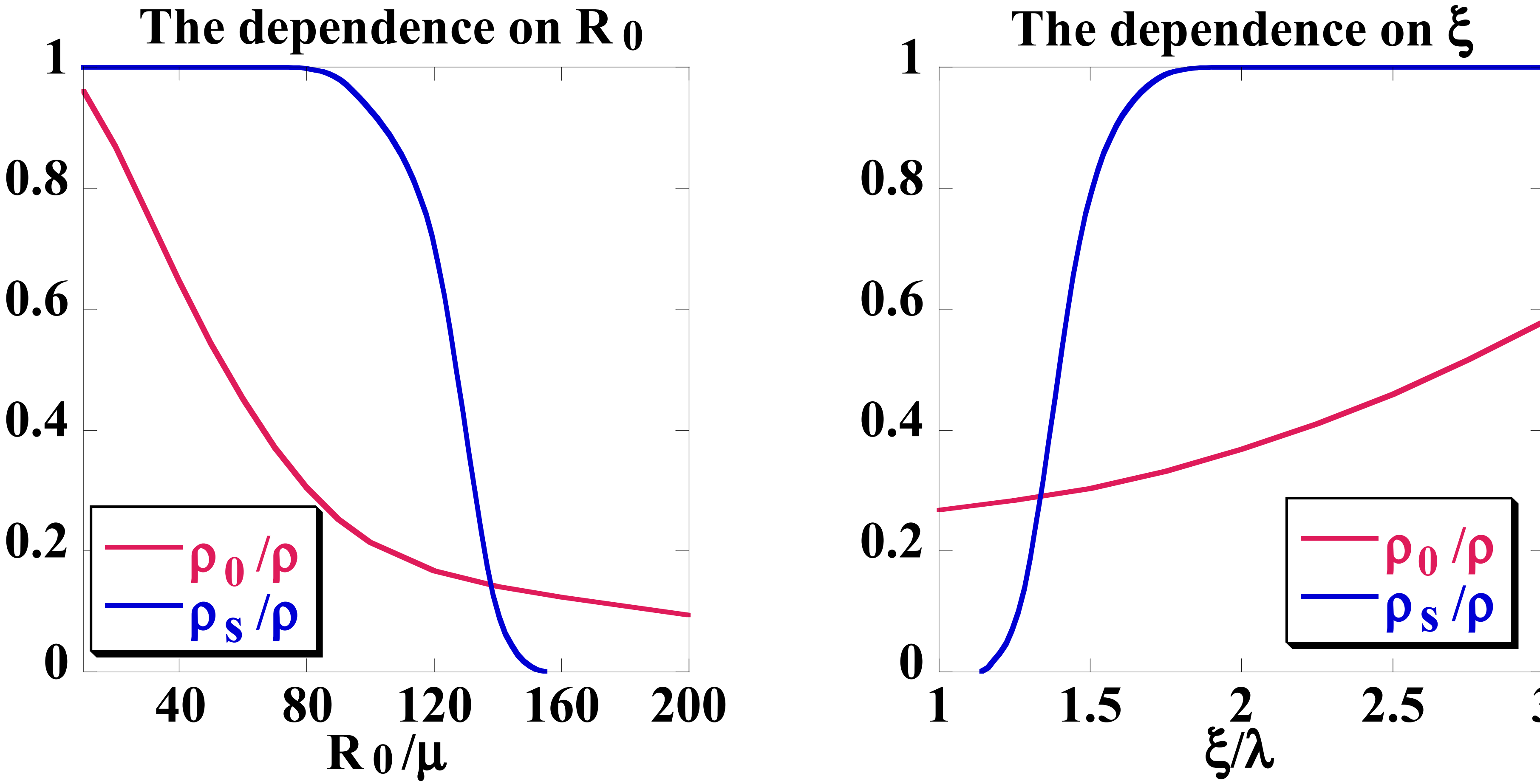
Wave function localizes as ξ decreases

Ground state depends on the shape of the random potential even at same parameters



The superfluid density of ground states

100 ensemble average at same R_0



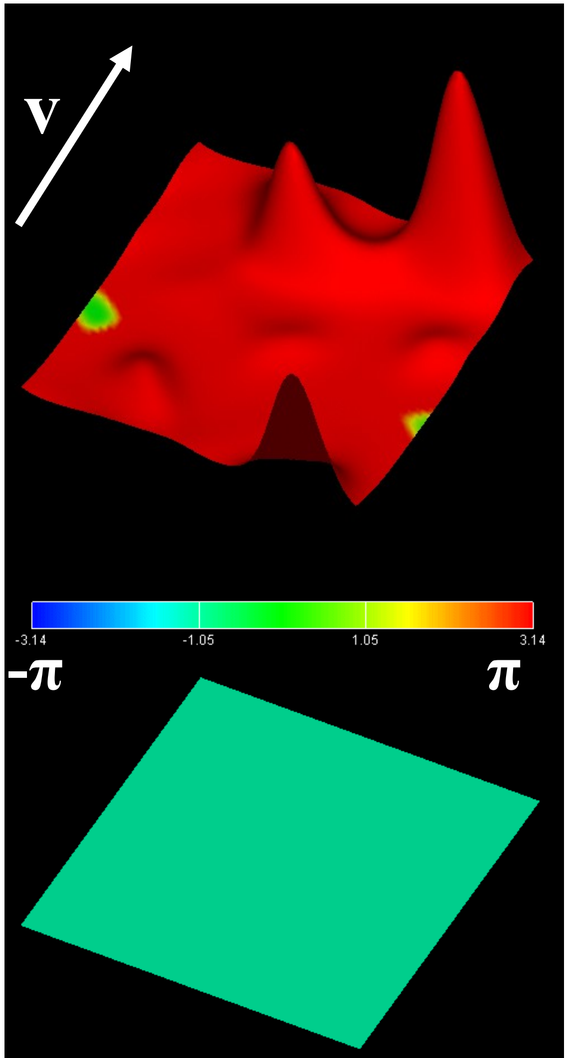
Superfluidity is depressed as the ground states localizes

Dynamics of vortex pairs in applied velocity field

amplitude

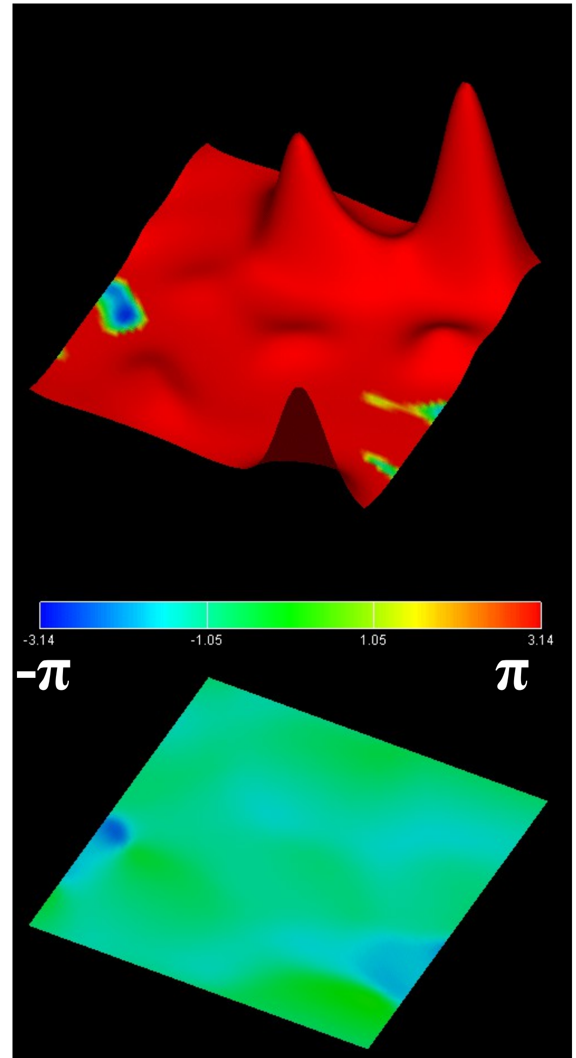
$$v\sqrt{\frac{2m}{\mu}} = 1.5$$

phase



$t\mu/\hbar = 0$

→



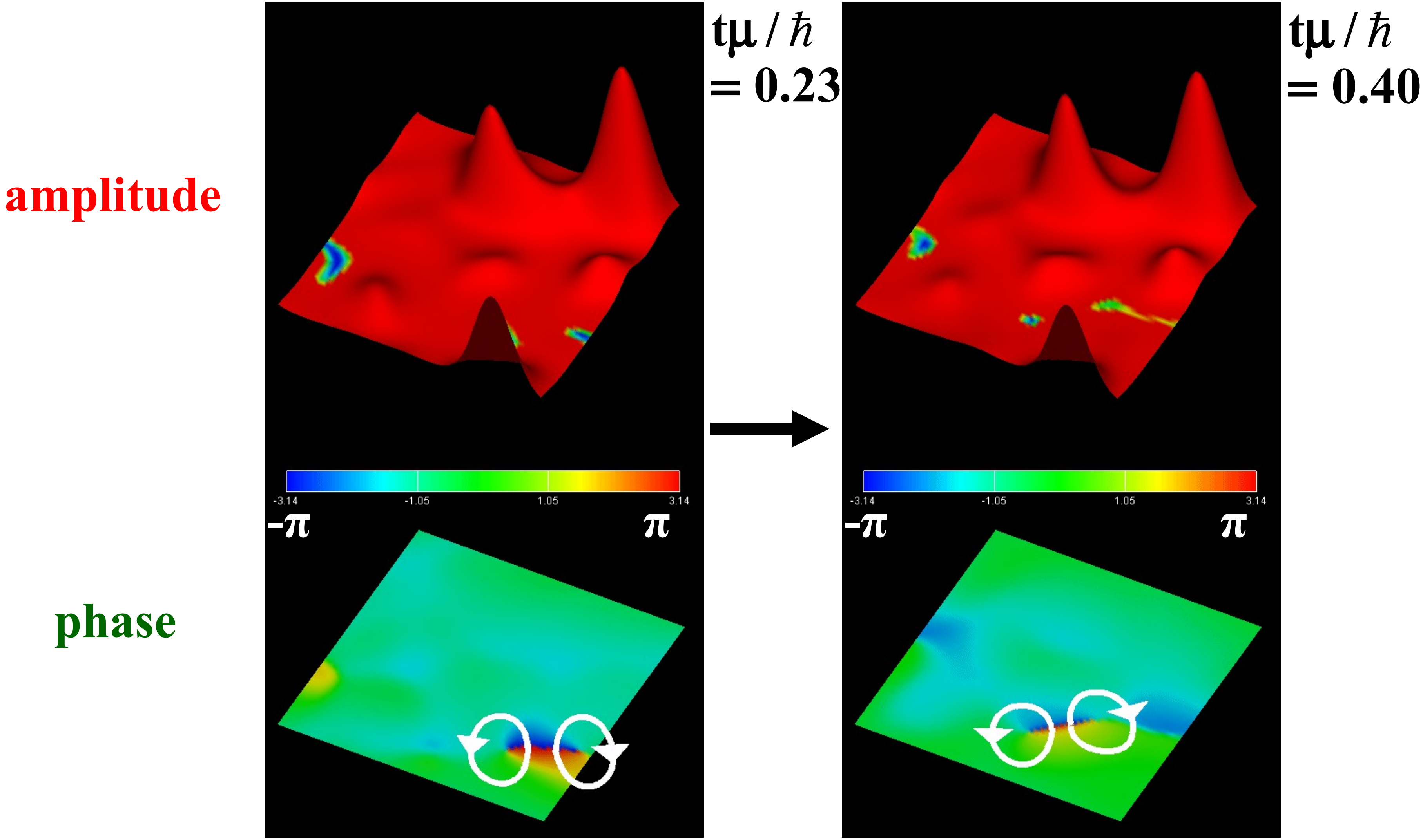
$t\mu/\hbar = 0.16$

→

$R_0/\mu = 50$

$\xi/\lambda = 2$

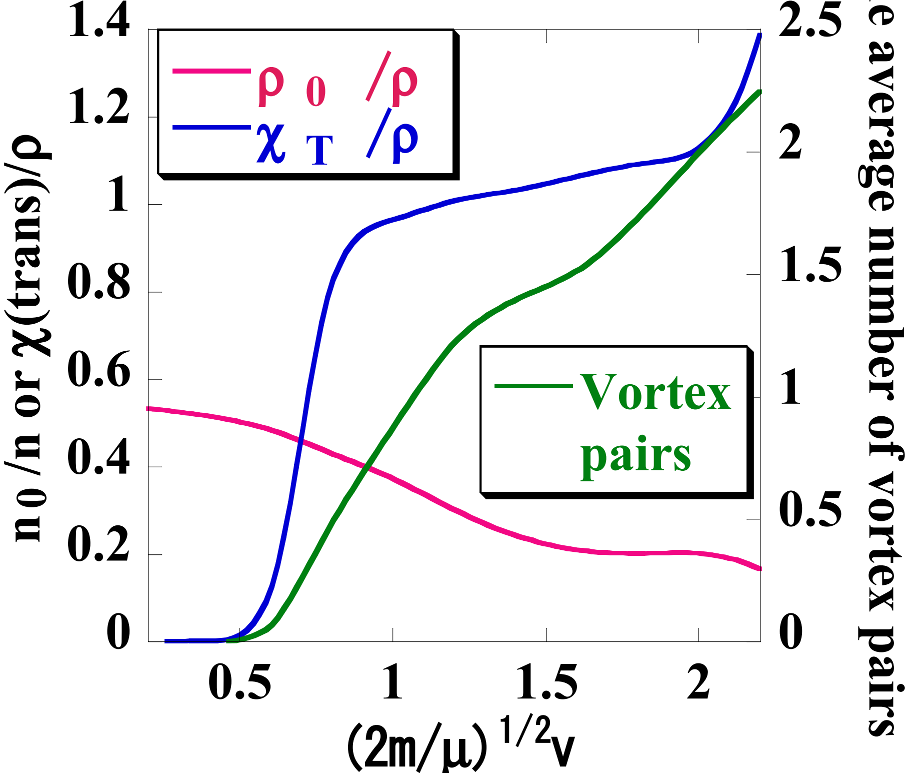
Phase becomes to be complicated



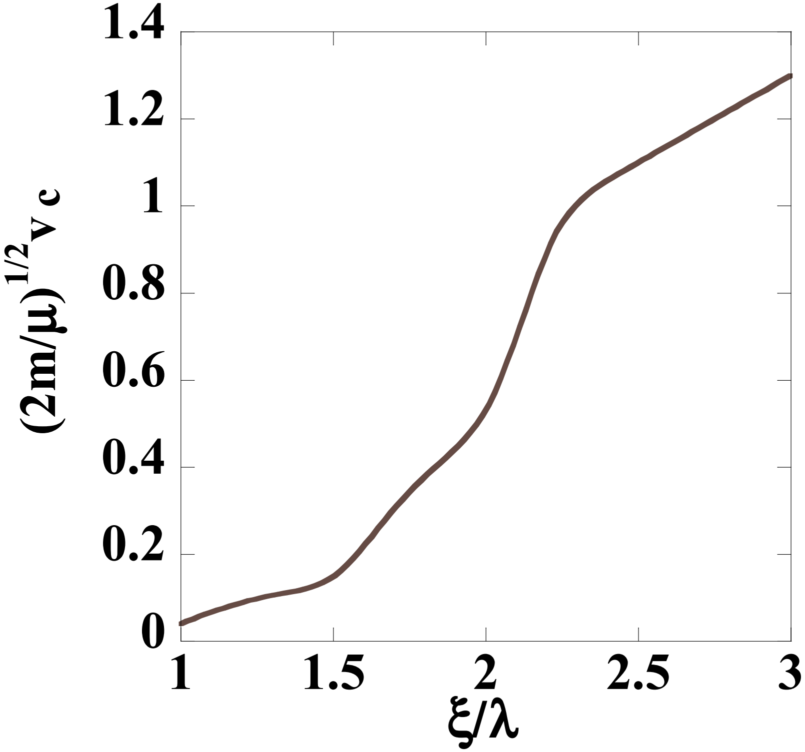
Vortex pair nucleates!
(Branch cut of the phase)

Dynamics of vortex pairs

The dependence on the velocity field



The dependence of the critical velocity on the coherence length



Nucleation of vortex pairs destroys superfluidity

The critical velocity is small as the wave function localizes

Conclusion

- By using GP equation with a random potential, the superfluid density in disordered system can be calculated.
- The dynamics of vortex pairs in applied field can be calculated
- We will expand this calculation to 3-dimension.