Dissipation of Gross-Pitaevskii Turbulence Coupled With Thermal Excitations

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Contents

1. Introduction -Kolmogorov spectrum in Gross-Pitaevskii turbulence-
2. Dissipation mechanism of quantum turbulence coupled with thermal excitations
3. Summary

1, Introduction -Kolmogorov Spectrum in Gross-Pitaevskii Turbulence-

M. Kobayashi and M. Tsubota, Phys. Rev. Lett. **94**, 065302 (2005).
M. Kobayashi and M. Tsubota, J. Phys. Soc. Jpn. **74**, 3248 (2005).

We successfully obtained the Kolmogorov spectrum in Gross-Pitaevskii equation with small-scale dissipation

GP equation

$$[\mathrm{i} - \gamma(\boldsymbol{x},t)]\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = [-\nabla^2 - \mu + V(\boldsymbol{x},t) + g|\Phi(\boldsymbol{x},t)|^2]\Phi(\boldsymbol{x},t)$$

$\Phi(\boldsymbol{x},t)$: BEC order parameter

$\gamma(\boldsymbol{x},t)$: Dissipation

μ : Chemical potential

$V(\boldsymbol{x},t)$: Energy injection

g : Coupling constant

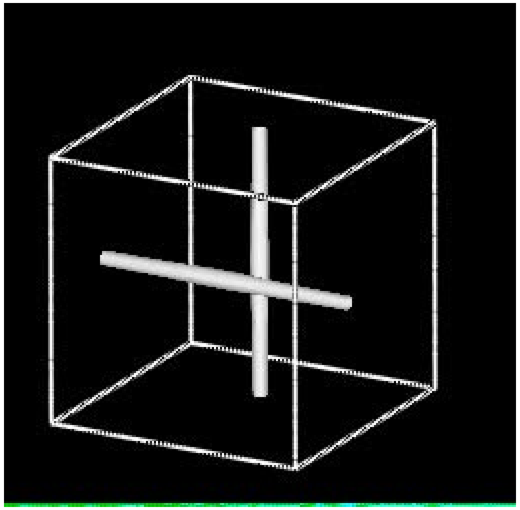
Small - Scale Dissipation

Fourier transformation

$$[\mathrm{i}-\gamma(k)]\frac{\partial}{\partial t}\Phi(\boldsymbol{k},t)=(k^2-\mu)\Phi(\boldsymbol{k},t)+\sum_{\boldsymbol{k}_1}V(\boldsymbol{k}_1,t)\Phi(\boldsymbol{k}-\boldsymbol{k}_1,t)$$
$$+g\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}\Phi(\boldsymbol{k}_1,t)\Phi^*(\boldsymbol{k}_2,t)\Phi(\boldsymbol{k}-\boldsymbol{k}_1+\boldsymbol{k}_2,t)$$

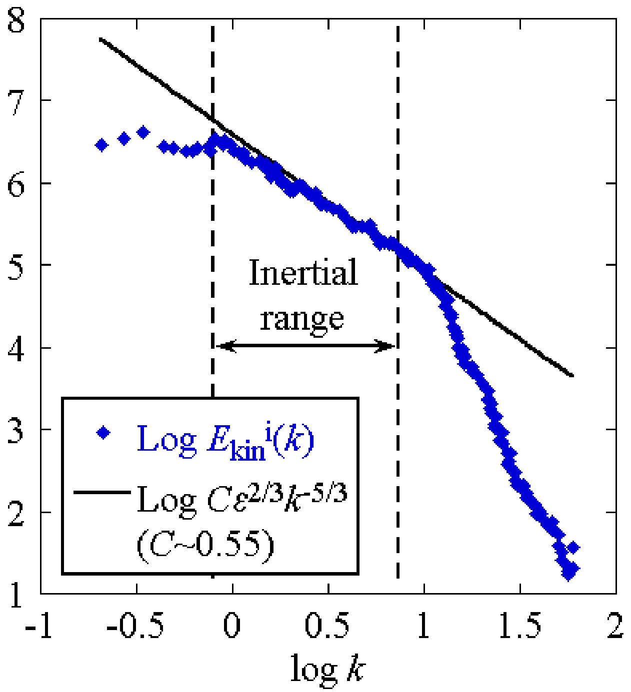
$$\gamma(k)=\gamma_0\theta(k-2\pi/\xi)$$

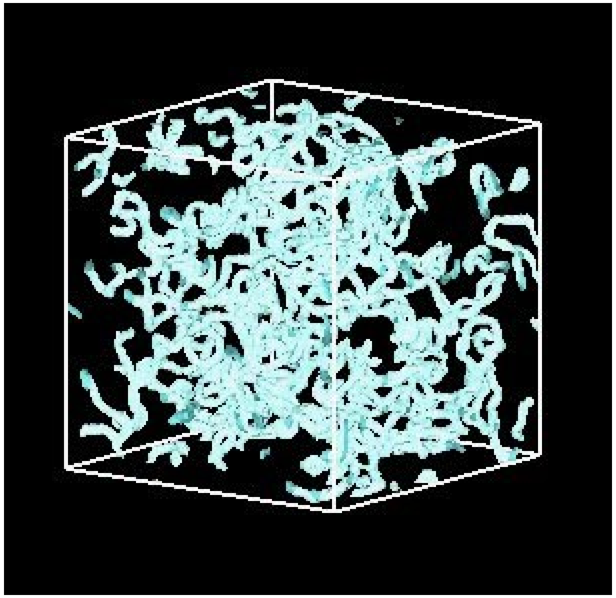
Small-scale dissipation that works only at scales smaller than the vortex core size



Vortex reconnection

Energy Spectrum in Quantum Turbulence





By dissipating short-wavelength excitations, we obtained the Kolmogorov energy spectrum

2, Dissipation of Quantum Turbulence Coupled With Thermal Excitations

What is the microscopic mechanism of dissipation ?
What is the realistic nature of introduced dissipation ?
How does dissipation change at finite temperatures ?

Is the form of dissipation

$$\gamma(k) = \gamma_0 \theta(k - 2\pi/\xi) \text{ correct?}$$

Mutual Friction in Superfluid Helium

For the case of superfluid helium

Quantized vortices dissipate through mutual friction between vortices and viscous normal fluid

$$\dot{\boldsymbol{x}}_0 = \alpha \boldsymbol{x}_0' \times (\boldsymbol{v}_{\rm n} - \boldsymbol{v}_{\rm s}) - \alpha' \boldsymbol{x}_0' \times [\boldsymbol{x}_0' \times (\boldsymbol{v}_{\rm n} - \boldsymbol{v}_{\rm s})]$$

How is the dissipation for the case of GP turbulence? (In dilute Bose gas)

Dissipation of GP Turbulence : Coupled System of GP and BdG Equations

Dissipation of GP equation can be discussed by considering the Bogoliubov-de Gennes equation of excitations

$$\hat{H} = \int \mathrm{d}\boldsymbol{x} \hat{\Psi}^\dagger(\boldsymbol{x}, t)[-\nabla^2 - \mu + \frac{g}{2}|\hat{\Psi}(\boldsymbol{x}, \boldsymbol{t})|^2]\hat{\Psi}(\boldsymbol{x}, t)$$

$$\mathrm{i}\frac{\partial}{\partial t}\hat{\Psi}(\boldsymbol{x}, t) = [-\nabla^2 - \mu + g\hat{\Psi}^\dagger(\boldsymbol{x}, t)\hat{\Psi}(\boldsymbol{x}, t)]\hat{\Psi}(\boldsymbol{x}, t)$$

$\hat{\Psi}(\boldsymbol{x}, t)$: Field operator of bosons

Dissipation of GP Turbulence : Coupled System of GP and BdG Equations

$$\hat{\Psi}(\boldsymbol{x},t) = \Phi(\boldsymbol{x},t) + \hat{\chi}(\boldsymbol{x},t) + \hat{\zeta}(\boldsymbol{x},t)$$

$$: \text{ Bogoliubov approximation}$$

$$\Phi(\boldsymbol{x},t) = O(\sqrt{N_0/V})$$

$$\hat{\chi}(\boldsymbol{x},t) = O(1/\sqrt{V})$$

$$\hat{\zeta}(\boldsymbol{x},t) = O(1/\sqrt{N_0 V}) \ : \text{ Neglect}$$

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2\langle \hat{\chi}^\dagger \hat{\chi} \rangle)]\Phi + g\langle \hat{\chi}\hat{\chi} \rangle \Phi^* \ : \text{ GP}$$

$$\mathrm{i}\frac{\partial \hat{\chi}}{\partial t} = [-\nabla^2 - \mu + 2g|\Phi|^2]\hat{\chi} + g\Phi^2 \chi^\dagger \ : \text{ BdG}$$

Effective Dissipation of GP Equation

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2\langle \hat{\chi}^\dagger \hat{\chi} \rangle)]\Phi + g\langle \hat{\chi}\hat{\chi} \rangle \Phi^*$$

GP equation has the imaginary part in Hamiltonian

$$\hat{H}_{\mathrm{GP}} = -\gamma(\boldsymbol{x}, t) = \mathrm{Im}\left[g \frac{\langle \hat{\chi}(\boldsymbol{x}, t)\hat{\chi}(\boldsymbol{x}, t)\rangle \Phi^*(\boldsymbol{x}, t)}{\Phi(\boldsymbol{x}, t)} \right]$$

Dissipation can be obtained naturally!

Calculation of Excitations

Bogoliubov transformation

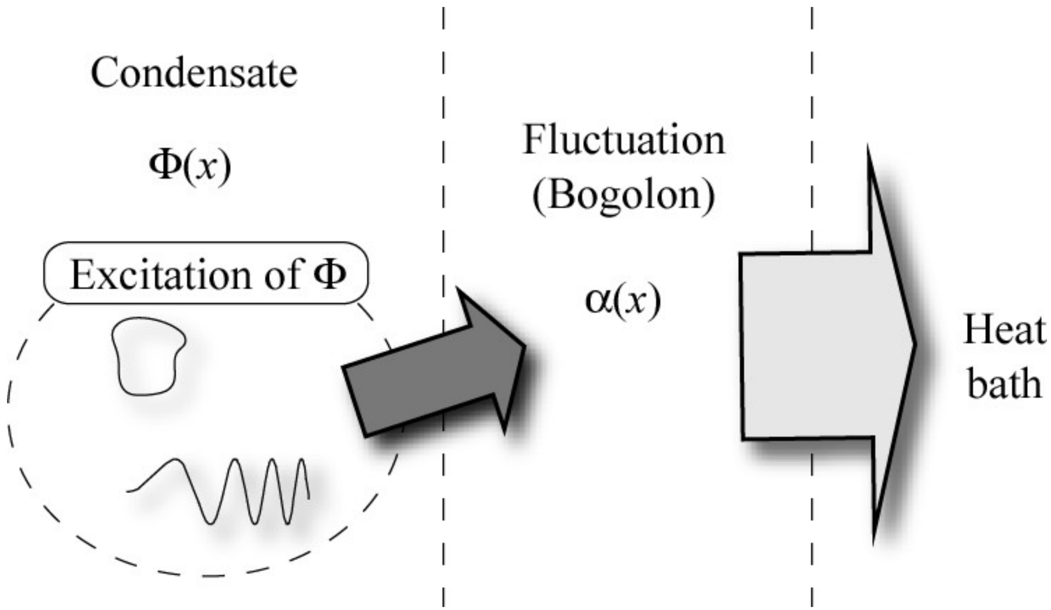
$$\hat{\chi}(\boldsymbol{x},t) = \frac{1}{\sqrt{V}} \sum_j \phi_j(\boldsymbol{x},t)\hat{a}_j = \frac{1}{\sqrt{V}} \sum_j [u_j(\boldsymbol{x},t)\hat{\alpha}_j + v_j^*(\boldsymbol{x},t)\hat{\alpha}_j^\dagger]$$

$\hat{\alpha}_j$ $(\hat{\alpha}_j^\dagger)$: annihilation (creation) operator of excitation bogolon

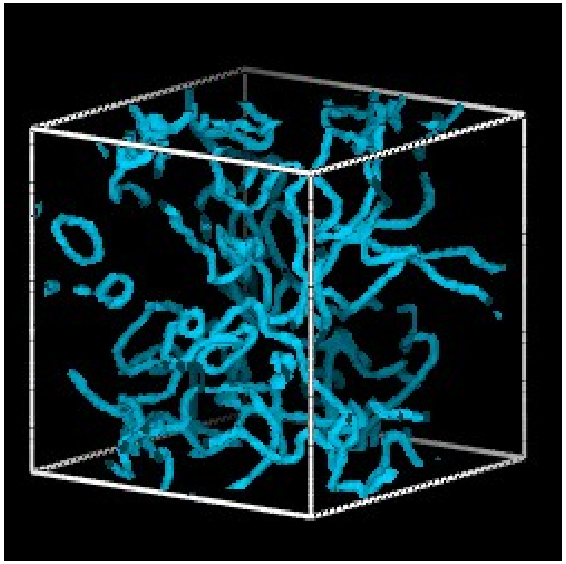
Calculation of Excitations

$$\langle \alpha_j^\dagger \alpha_j \rangle = N_j = \frac{1}{\exp[E_j/T] - 1} \ : \ \text{Local equilibrium approximation}$$

: Bogolons are coupled with the heat bath



Initial State for Numerical Simulation



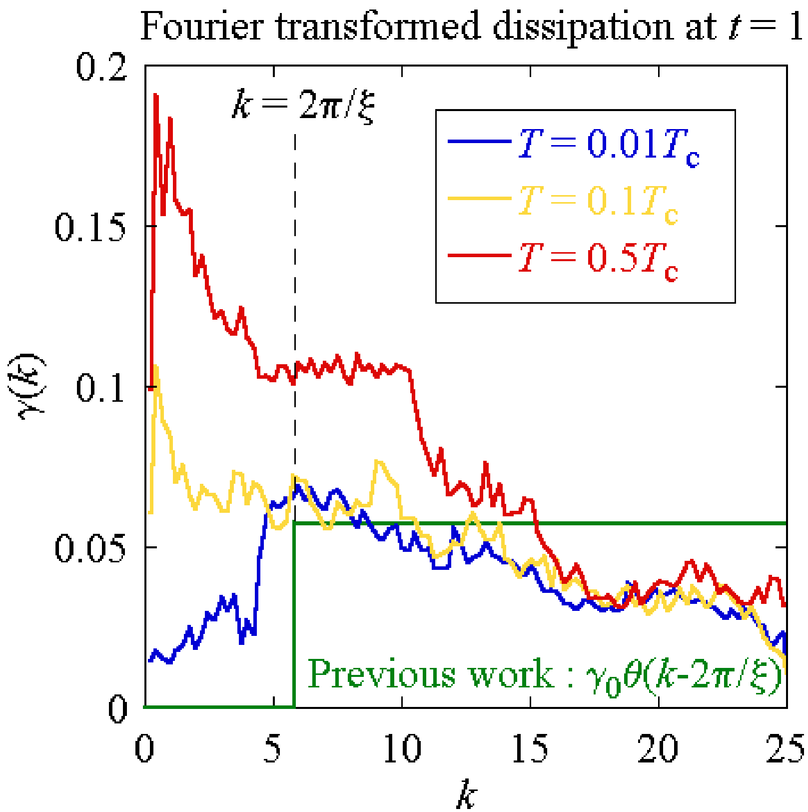
Condensate : randomly placed some vortices

$$u_j(\boldsymbol{x}, t=0) = \mathrm{e}^{\mathrm{i}\boldsymbol{k}_j \cdot \boldsymbol{x}} \sqrt{\frac{1}{2V} \frac{k_j^2 + g|\Phi|^2}{E_j} + 1}$$

$$v_j(\boldsymbol{x}, t=0) = \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_j \cdot \boldsymbol{x}} \sqrt{\frac{1}{2V} \frac{k_j^2 + g|\Phi|^2}{E_j} - 1}$$

Excitation : uniform solution

Numerical Result : Dissipation Term



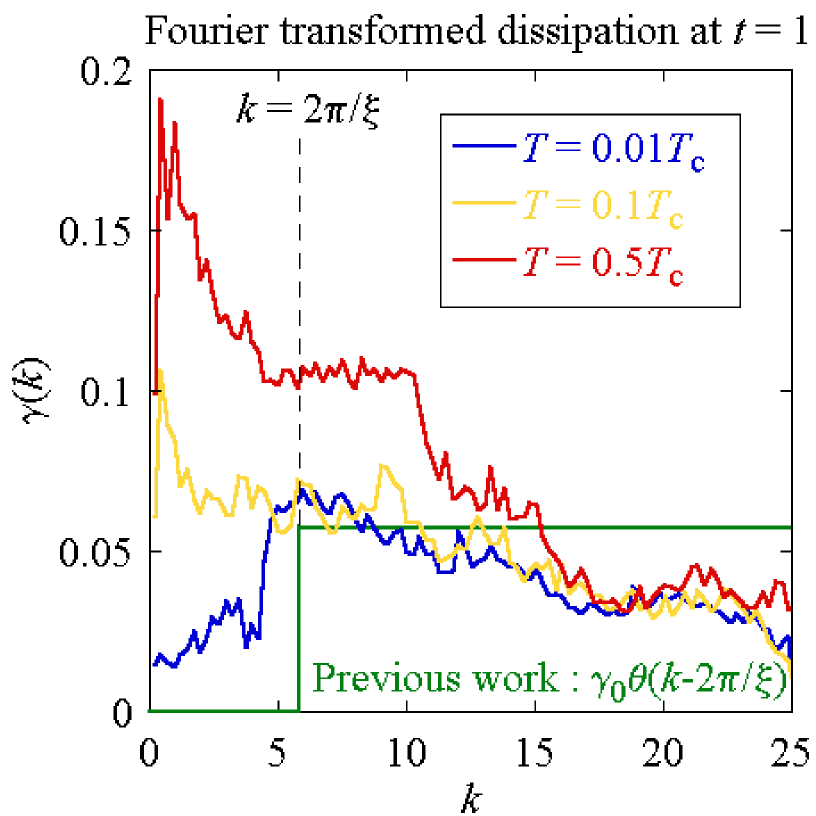
$T_c = 4\pi/\{\zeta(3/2)\}^{2/3}$: Critical temperature of the BEC of free bosons

At low temperature :

Dissipation works at scales smaller than the healing length and consistent with the dissipation introduced in our previous work

→Only short wavelength excitations are dissipated

Numerical Result : Dissipation Term



$T_c = 4\pi/\{\zeta(3/2)\}^{2/3}$: Critical temperature of the BEC of free bosons

At high temperature :

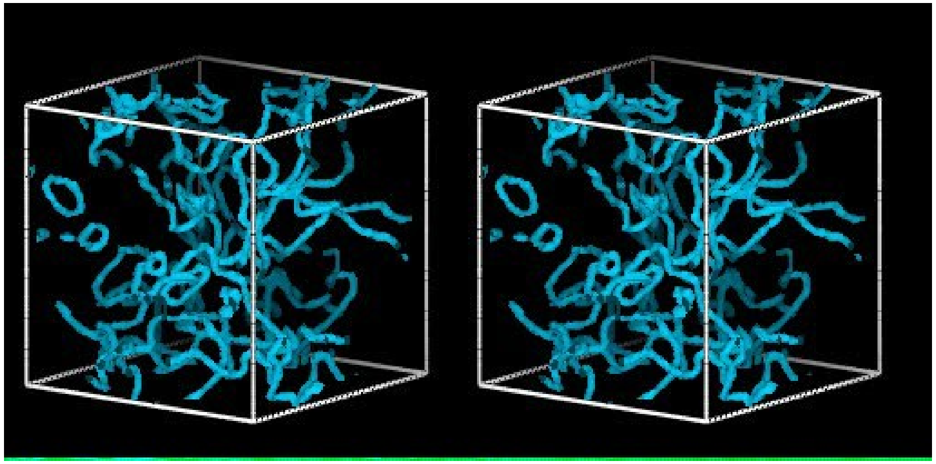
Dissipation works at large scales as well.

→Vortices are dissipated and vortex dynamics is affected by the dissipation

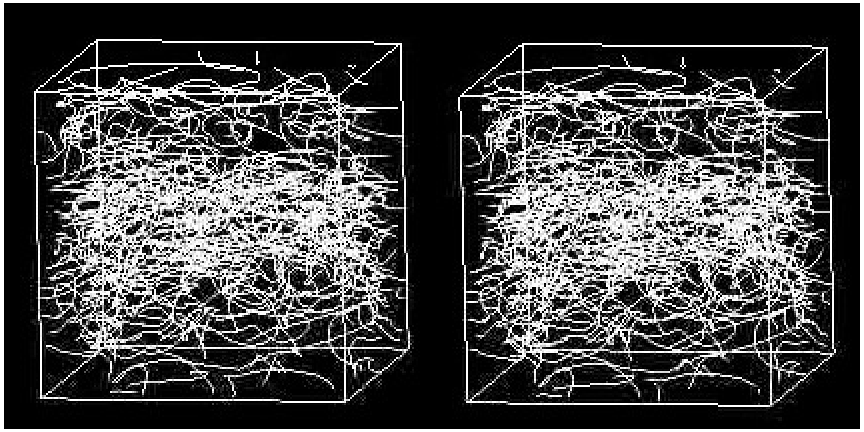
→Similar to mutual friction

Numerical Result : Vortex Dynamics

$T = 0.01T_\mathrm{c}$ $T = 0.1T_\mathrm{c}$



with without
mutual friction



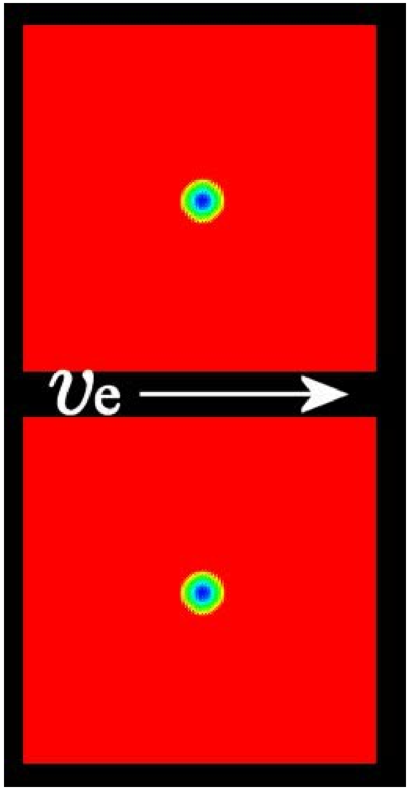
M. Tsubota, T Araki and S. K. Nemirovskii, Phys. Rev. B **62**, 11751 (2000).

Vortices are more dissipated (including Kelvin wave) at higher temperatures (Similar to simulation by the vortex filament model).

Comparison With Mutual Friction

Dynamics of 1 straight vortex (2D simulation) under the velocity field

$T = 0.01T_c$



$T = 0.1T_c$

$$\dot{\boldsymbol{x}}_0 = \alpha \boldsymbol{x}_0' \times \boldsymbol{v} - \alpha' \boldsymbol{x}_0' \times [\boldsymbol{x}_0' \times \boldsymbol{v}]$$

Drag force

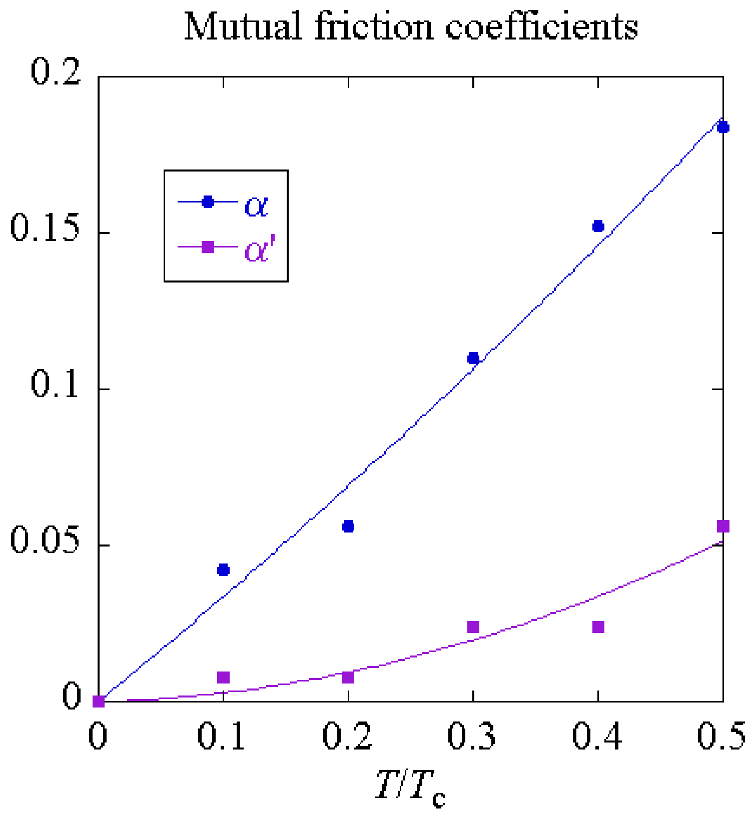
$$\boldsymbol{v} = [v_e, 0]$$

$$\boldsymbol{x}_0(t) = [x_{0x}(0) + \alpha' v_e t, x_{0y}(0) + \alpha v_e t]$$

$\boldsymbol{v}_e$: Velocity field

Moving flame with $\boldsymbol{v}_e$

Comparison With Mutual Friction



We successfully calculate the mutual friction coefficients for the case of GP turbulence

→ need to be experimentally observed in dilute BECs (Is impossible to directly compare with those in ^{4}He)

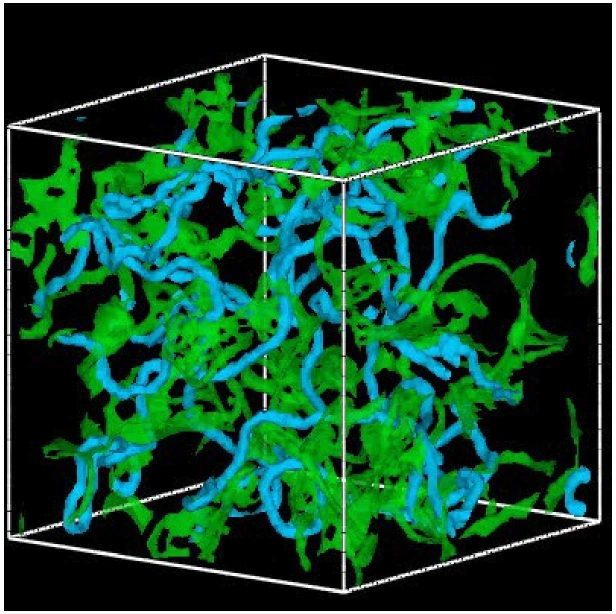
Coupled Turbulence

W. F. Vinen, Phys. Rev. B **61**, 1410 (2000).

In superfluid helium, superfluid and normal fluid are likely coupled together at large scales due to mutual friction and behave similar to the turbulence in a one-component fluid

We can expect a similar coupled turbulence in which the dynamics of thermal excitations is coupled with that of the condensation and both the dynamics become comparable at large scales.

Coupled Turbulence



$T = 0.1T_{\rm c}$

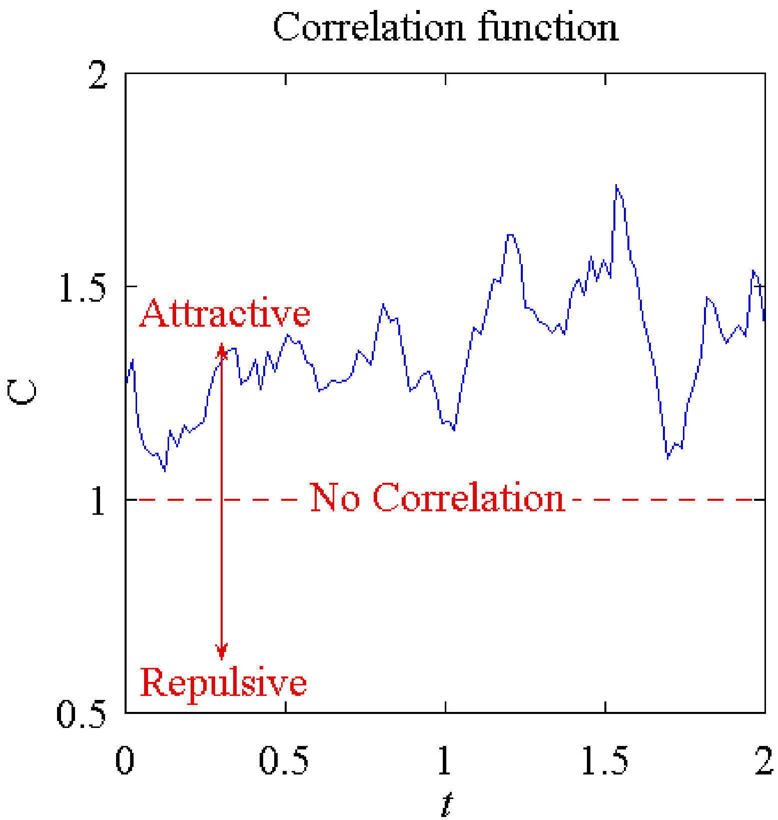
Blue : Quantized vortices

Green : Region of high vorticity of noncondesate

$$|\boldsymbol{\omega}_{\rm e}(\boldsymbol{x})| > 0.95\ \langle|\boldsymbol{\omega}_{\rm e}|\rangle$$

We can see highly tangled turbulence made of quantized vortices and noncondensate eddies

Correlation Between Quantized Vortices and Noncondensate Eddies



- Correlation function is always larger than 1 : Quantized vortices and noncondensate eddies are attractive.

We confirm the signal of coupled turbulence!

3, Summary

1. We calculate the coupled system of GP and BdG equations and investigate the microscopic mechanism of the dissipation in quantum turbulence.
2. At low temperatures, dissipation works only at scales smaller than the vortex core size, which is consistent with the dissipation introduced in our previous work.
3. At high temperatures, dissipation works at large scales as well and directly affect the vortex dynamics.
4. We successfully relate the dissipation at high temperature with mutual friction in superfluid helium by calculating the mutual friction coefficients as functions of temperature.

Quantized Vortex in GP Equation

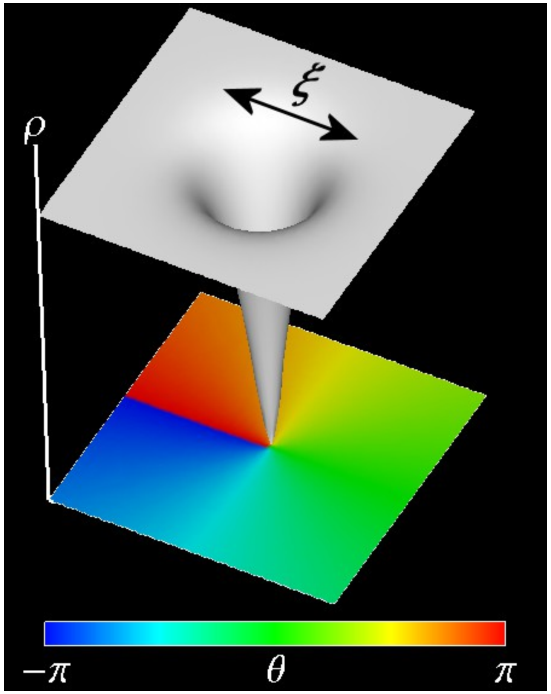
$$[\mathrm{i} - \gamma(\boldsymbol{x}, t)] \frac{\partial}{\partial t} \Phi(\boldsymbol{x}, t) = [-\nabla^2 - \mu + V(\boldsymbol{x}, t) + g|\Phi(\boldsymbol{x}, t)|^2] \Phi(\boldsymbol{x}, t)$$

$$\Phi(\boldsymbol{x}) = |\Phi(\boldsymbol{x})| \exp[\mathrm{i}\theta(\boldsymbol{x})]$$

$$\rho(\boldsymbol{x}) = |\Phi(\boldsymbol{x})|^2 : \text{ Density}$$

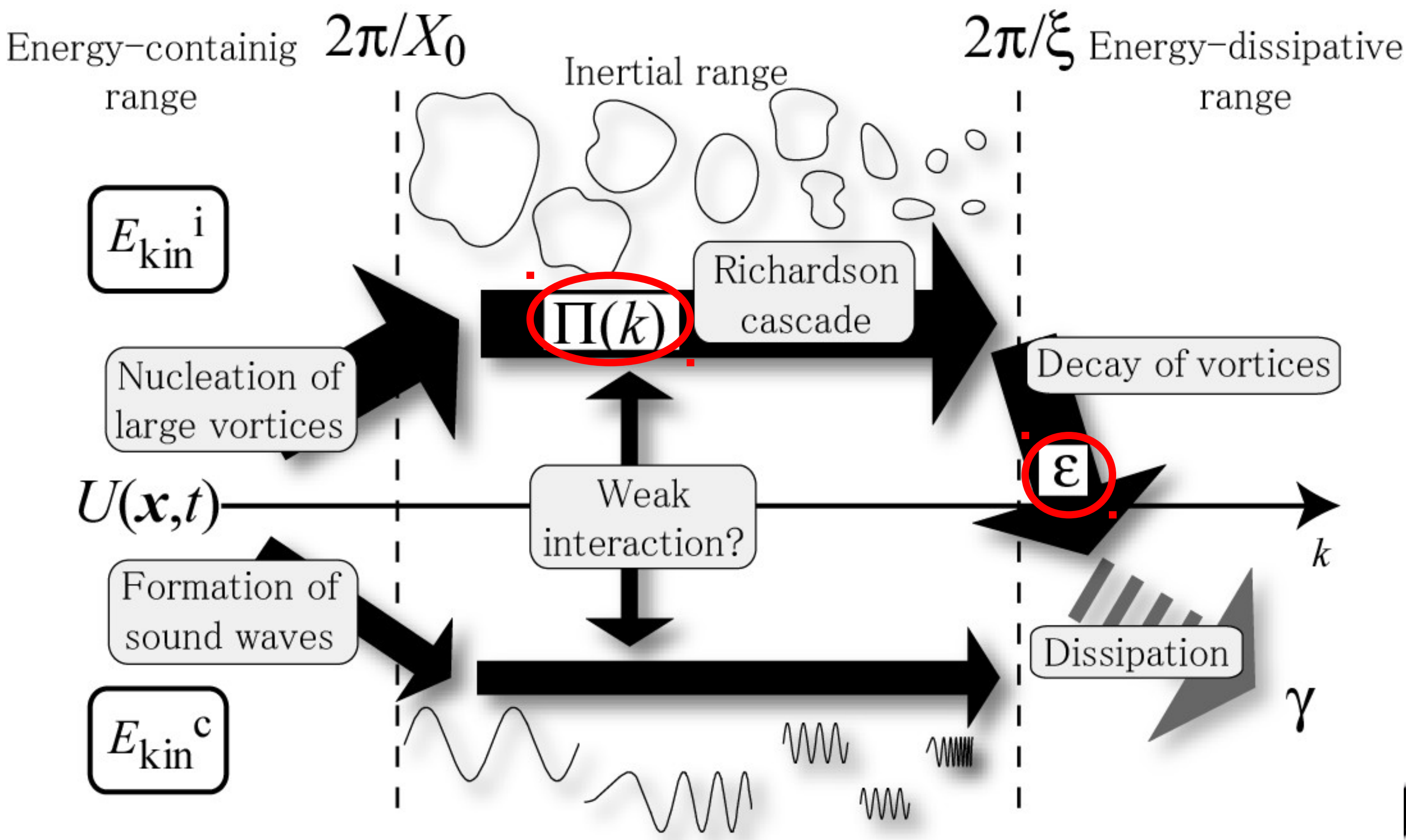
$$\boldsymbol{v}(\boldsymbol{x}) = 2\nabla\theta(\boldsymbol{x}) : \text{ Velocity field}$$

$$\xi = 1/\sqrt{g\bar{\rho}} : \text{ Healing length}$$



Quantized vortex

Flow of Energy in Quantum Turbulence



Final Form of Equations

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2n_{\mathrm{e}})]\Phi + g m_{\mathrm{e}} \Phi^*$$

$$\mathrm{i}\frac{\partial u_j}{\partial t} = [-\nabla^2 - \mu + 2g|\Phi|^2]u_j - g\Phi^2 v_j = A_j$$

$$\mathrm{i}\frac{\partial v_j}{\partial t} = -[-\nabla^2 - \mu + 2g|\Phi|^2]v_j + g\Phi^{*2} u_j = B_j$$

$$n_{\mathrm{e}} = \sum_j [|u_j|^2 N_j + |v_j|^2 (N_j + 1)] \,:\, \text{Noncondensate density}$$

$$m_{\mathrm{e}} = -\sum_j [u_j v_j^* (2N_j + 1)]$$

$$E_j = \int \mathrm{d}\boldsymbol{x}\ \mathrm{Re}[u_j^* A_j + v_j^* B_j] \,:\, \text{Excitation spectrum}$$

Simulation Parameters

$$N = 32^3 \text{ grids :}$$

$$g = 1 \qquad \Delta x = 0.125 \qquad V = 4^3 \qquad \Delta t = 5 \times 10^{-4}$$

Correlation Between Quantized Vortices and Noncondensate Eddies

Correlation function : $C(t) \equiv \dfrac{L_{\mathrm{e}}/L}{\int \mathrm{d}\boldsymbol{x}\, p_{\mathrm{e}}(\boldsymbol{x})/V}$

L : Total line length of quantized vortices

L_{e} : Total line length of quantized vortices in the region of $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95 \langle |\boldsymbol{\omega}_{\mathrm{e}}| \rangle$

V : Total volume

$p_{\mathrm{e}}(\boldsymbol{x}) = 1$ if $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95 \langle |\boldsymbol{\omega}_{\mathrm{e}}| \rangle$

$p_{\mathrm{e}}(\boldsymbol{x}) = 0$ if $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95 \langle |\boldsymbol{\omega}_{\mathrm{e}}| \rangle$

$C(t) < 1$: Vortices and eddies are repulsive

$C(t) = 1$: No correlation between vortices and eddies

$C(t) > 1$: Vortices and eddies are attractive